

Piketty's Prediction meets technical progress in Harrod-Domar's Dynamics and Solow-Swan's Surrogate

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Comments are invited

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Abstract

Thomas Piketty [2014] uses mainly UN projections that the world *GDP* growth rate g can fall by half in 2010-2100, to predict that the capital or wealth share in world national income, α_K , may double. This prediction for α_K contradicts at least one of Kaldor's stylized facts and possibly the Kuznets curve. Negating somewhat this prognosis, we show here formally, that exogenous technical progress can vary the α_K trajectories in either direction, even with the saving rate, s , held constant. Further, we show how s can be used not only to stabilize α_K but also to allow *GDP* growth g to accompany its only true engine, technical progress. Like Piketty, we use the Solow-Swan model and ignore the recent literature which, since about 1985, tries to make endogenous this mother lode of growth. Finally, the paper suggests that the policy recommendation of this literature, for the great majority of the world's population, should continue to emphasize growth in *per capita GDP*, at least while the gap with the rich economies is so wide. The issue remains income distribution, not the ones within each economy, Piketty's contribution, but rather the one across them all.

Preface to Version 2.2 October 2015

This revision keeps the paper's focus on the effect of technical progress on the capital share, α_K , and not so much on the other central issue of the book, Piketty's perhaps more famous $r > g$ analysis; famous in that folks now wear Tee shirts with $r > g$ printed on their chests. I, like several other authors, did not find compelling the $r > g$ link to his projected increases in inequality in the distribution of inherited wealth in 21st century capitalism e.g. Acemoglu & Robinson [2014], Charles I. Jones [2015]. Since my reasons differ somewhat from these folk, I thought it useful to include them here.

The issue begins with a surprising find from Piketty's own data. For most of the 20th century, 1913-2012, the **after-tax** rate of return on capital, \underline{r} was always **less than** the growth rate, g (Fig. 10.10 p.356). Maybe the Tee shirts should read $\underline{r} < g$. Before 1914, taxation of incomes was virtually non-existent and so $r \approx \underline{r}$. In the 20th century, however, taxes on wealth in all its forms altered the behavior of the rich. Inheritance and bequests now required new theories and models. Piketty and his researchers used them, but for a book aimed at a wider audience, he had to limit the exposition. The technical work on inheritance is mostly in their papers.

The topic is specialized and better treated in a separate study. It is an important one and should be pursued further. Here I examine briefly the economic incentives and stochastic processes it introduced – random bequest motives and demographic dynamics. I call attention especially to the new Appendix III but there are several other innovations in this Version 2.2. Where to find them is indicated on the title page.

In addition to Piketty's Harvard UP book, I consulted its 97 page online Technical Appendix which not only serves as an excellent guide on where to find the various topics in the book, but also allows downloads of the relevant journal articles. A quick introduction to his ideas is his 36 slides from a lecture at Helsinki, Finland in November, 2013. The book reviews by several outstanding economists were also very helpful. All of these materials, except the book, are available here, open access online, at our clickable bibliography. There is also a traditional non-clickable one. The Brazilian edition preserves the numbering of the tables and figures but not the pagination e.g. Fig.10.5 remains the fifth figure in Chapter 10.

Piketty's Prediction meets technical progress in Harrod-Domar's Dynamics and Solow-Swan's Surrogate

Steve De Castro October 2015

Let's begin by stating what we think is the central drama of Piketty's book, his prediction for the world economy in the 21st century:

Given that the world *GDP* growth rate g is projected to fall by half after 2050, then the capital or wealth share in world national income, α_K , may double by 2100.

It seems dismissive if not foolhardy to sum up a volume of over 600 pages in one sentence, given the elegant empirics with which Piketty supports the Prediction, and how he hedges it with many caveats and other possible scenarios. Two aspects of the book gave us the cue for the drama. One is that, while his prognosis is for the whole world, the long series he studies are data for mainly a few large, currently rich economies, and especially Britain, France and the US. The other is that unlike most academic economists in this field, Piketty completes his exposition with a polemical policy recommendation; an initial small world tax on capital incomes which he suggests may help governments to monitor this explosion in the capital share. However, in this paper our interest lies, as its title implies, more with the economic theories which inspired his careful empirical work and which led him to the Prediction. Specifically, our main contribution is to introduce the role of technical progress, obscured somewhat not only in the book but even in the more formal analyses of some reviewers.

In the Prediction, these potentially massive increases in the capital share, α_K , follow from what Piketty called the first two Fundamental Laws of Capitalism. A separate major topic in the book, but not central to this paper, is an expected widening in the world economy of the gap between the rate of return to capital (r) and the *GDP* growth rate (g) (Fig.10.9 p.354), and its possible future consequence of even further inequality in the distribution of accumulated wealth. In both research topics there is no treatment of technical progress, the dominant cause of the Great Divergence of *GDP* per capita across economies. Another common element is the assumed trajectory of a much lower world g after 2050. Some reviewers have dubbed this $r > g$ prognosis his Third Law (e.g. Debraj Ray [2014]). Piketty called it the Central Contradiction of Capitalism (last Chapter, Conclusion, not numbered, p.571). There are several variants in the book. Here is our succinct version:

If the gap ($r - g$) widens as g falls drastically after 2050, then inherited wealth can return to the dominant share of total wealth it had before 1914, and can be just as unequally distributed.

The world economy may return to the state of "*patrimonial capitalism*" in which it was before 1914, when the wealthiest saw no need to work - the idle rich (Piketty [2014] Chap.7 pp.238-40, Vaudrin's Lesson from Balzac's *Pere Goriot*). Piketty showed that when the gap ($r - g$) widens, the theoretical shape of the expanding wealth distribution becomes a Pareto power law in which the top 10% and 1% hold most of it (Chap.10 pp.364-5). This research, however,

mobilizes quite different data from the α_K models, both his and ours, and is better treated in a separate paper. It postulates the existence of individual inter-temporal preferences such as bequest motives, and uses estimates for the birth and death rates of the rich to project the theoretical, long-run equilibrium distribution across holders of wealth (Piketty [2014] online Technical Appendix, p.60-2). Some analyses of Piketty's $r > g$ models have been done e.g. Acemoglu & Robinson [2014], and Charles I. Jones [2014], also available online in our clickable bibliography, and Charles I. Jones [2015] cited in our traditional one. These authors are not convinced by the $(r - g)$ modeling. For more details, see Appendix III here.

This topic in the book is perhaps both the most innovative work and yields its least compelling insights. For example, Piketty's long-run data show that r , the world rate of return before tax, has remained above g from “antiquity” (year zero BC, AD, CE) to 2012 (Fig.10.9 p.354). Yet, for most of the 20th century, from 1913 to 2012, the after-tax return has remained below g . It is true that these GDP growth rates were much higher than the extremely low ones which prevailed before the industrial revolutions (Fig.10.10 p.356). Nevertheless, the net return to the capital of private agents was so drastically reduced by taxation that it must have induced structural change in the dynamics of inherited wealth. Another example is the US case from which emerged the odd theoretical result that any future decrease in population growth can cause inherited wealth to “rebound as strongly there as in Europe” (Chap.11 p.428).

Our paper is not a book review. A few comments are offered now partly to prep those who may not have read it, but also to allow a quicker grasp of the 9 Points set out here. Some who are familiar with the earlier results of Harrod-Domar, and standard Solow-Swan, that is, without technical progress, may want to read up to and including Point 1, and then to skip to Points 5 and beyond. For those who shouldn't bother to spare the time to read the book, Piketty's online Technical Appendix and the many reviews cited here can provide useful synopses.

Firstly, Piketty's data and focus are mainly on the functional distribution, especially the share of property income in the GDP for these rich economies. He also studies the long-term trends in the top of the personal income distribution, the 1% and 10% richest households. In the US case, however, over 60% of the increased share since about 1980 of the top 10% is due to labor incomes – the super-salaries of the few, so that rising capital incomes there are **not** its main component (Piketty [2014] Chap.9 pp.315-7; see also Krugman [2014] *NY Review* and Solow [2014] *New Republic* in our clickable bibliography).

Since the 1980ties, the US, followed somewhat by the “Anglo-Saxon” economies, reduced drastically the progressivity of the taxes on labor incomes. Some marginal rates were as high as 90%. Few ever paid them. All forms of tax avoidance were practiced, generating huge fees for tax lawyers. Piketty (Chap.9 pp.330-3) questions whether the subsequent super-salaries are based on any observable measure of higher productivity as these economies have not grown

any faster since the reform. Krugman [2014] thinks they are market values, especially in the finance sectors where labor productivity is usually observable. However, it is doubtful whether these super salaries should be treated as labor incomes since they bear little resemblance to those normally paid to workers with similar professional qualifications. We suggest that both before and after the tax reform, these high rewards to labor may be a social inefficiency caused by information asymmetries both within the finance oligopolies, and between them and their regulators. The 2007 US finance crisis is a symptom. The driving incentives, of course, are the huge returns to mainly US and UK finance firms after the collapse of the Bretton Woods doctrines freed international financial flows (see Point 9).

Secondly, although Piketty's Prediction for 21st century capitalism assumes crucially a projected long-term stable growth rate, his g is the growth rate of GDP and not the GDP per capita ($GDPpc$) one studies in growth theory. This can be confusing to some readers (and reviewers) because only economists who study business cycles focus on GDP since, in the very short-run, population changes are usually trivial and largely irrelevant.

Thirdly, since both his empirics and our theoretical work are posed around a measure of the total income and total output of a closed economy, Y here, we need to be clear about their definitions. From the national accounts, all three of the following, $(rK + wL)$, I/s and $F(K,L)$, must yield the same measure, Y . For example, the Keynesian multiplier, $Y = I/s$, where $s \equiv S/Y$, the saving ratio, reaches Y from the expenditure side, $Y = C + I$. To be consistent with Y given by $(rK + wL)$, the income side, the definitions of I , investment, and K , property or wealth, must be compatible. Specifically, if both I and K are measured net of depreciation, then in a financially closed economy Y becomes National Income, not GDP . This may affect the calculation of both the GDP growth rate, $g \equiv \Delta Y/Y$, and since $I = S$, also the saving ratio, s . The long-term trends of these two parameters g and s are central to Piketty's Prediction.

Finally, our Points 1 to 8 are based on theories for a closed economy and thus are not particularly relevant for the open ones of the 21st century. However, the 15 to 20 largest countries, not all poor, contain over 80% of the world's population. When compared to the other 200 or so in the UN list, these mega-economies can be treated for many purposes as effectively closed. For example, in $Y = C + I + X - M$ for a large open economy, one can argue that X, M are both relatively small, and that $X \approx M$ most of the time, so that $Y \approx C + I$ is not an unreasonable approximation, whether X, M are goods or services (including factor payments e.g. interest). Then all the results here for Harrod-Domar and Solow-Swan go through. In any case, these are the growth models which seem to have inspired him. Only briefly, in Point 9, do we draw attention to the financially open economies which have emerged in the 21st century.

Point 1 Piketty's Prediction unveiled

To begin, we distinguish between his Prediction for α_K and the mainly UN projections of world GDP growth rate, g , for the 21st century which Piketty used to make it. These

projections showed that both g and those for the world population, n , are expected to fall continuously after 2050 and to stabilize at around half their values sometime before 2100 (Piketty [2014] Fig.2.5 p.101; Fig.10.10 p.356; Helsinki lecture slide 17). Piketty used these UN projections and his own discovery that the (pre-tax) rate of return on capital, r , for the world economy had hardly changed since “*antiquity*” (Fig.10.9 p.354), to make his Prediction for α_K . We show now how he does this.

The capital share in national income is defined as $\alpha_K \equiv rK/Y$, $0 < \alpha_K < 1.0$, where r is the rate of return on capital, K , net of depreciation. Since Solow, it is conventional to include in rK all forms of non-labor income and, for his Prediction, so does Piketty. He also calls K , wealth, and measures it in the same current value terms as Y . So for example, if he finds $K/Y = 4.5$ then, since Y is the total current output in one year, he would say that the wealth of this economy is worth 4.5 years of current output. In 2010, K/Y was about 6.0 in France (Fig.3.6 p.128) and 5.0 in UK (Fig.3.5). In 1910, these were both 7.0 since 1700. Between 1914 and 1974, K/Y fell significantly to less than 3.0 in both before taking off again. This take-off is extrapolated to 2050 and 2100 and extended to the world economy to yield his Prediction. In the US, K/Y was more stable in the 20th century, varying between 4.0 and 5.0 (Fig.4.8 p.154).

In his 20th century data, Piketty refused to separate out human capital from labor and to include it in the other forms of capital (Chap.1, p.46), on the grounds that only in a slave society can human capital be owned by another person. With this logic, he included slaves in the US capital stock. In 1770, for example, slaves constituted about one third of total US wealth, about the same as the value then of agricultural land. By 1880, his graph showed it was zero (Piketty [2014] Fig.4.10 p.160; Helsinki lecture, slide 24). One explanation for this apparent contradiction is, as we saw above, Piketty’s interest in a capital stock which is inheritable. Clearly, abolition destroyed one version of inheritability apparently without transferring this wealth to the former slaves, in contrast to when, at the French Revolution, agricultural land was redistributed to the former vassals of the aristocracy and the Church, a land reform which the English never needed or dared.

We claim that whether a work force is in some sense inheritable in both societies is determined by the type of labor contract each legitimizes. Piketty admits that all modern legal systems allow a worker to offer his services for hire “*under a labor contract of some sort ... (provided it is) limited in time and scope*” (Chap.1 p.46). A labor contract, it seems, can be a substitute for ownership of the worker. Think of the value to investors in modern football clubs, and the clubs’ contracts with the players. In our research on the microeconomic theory of the transition from modern slavery to the employment contract, we identified the right to quit as the crucial difference, and not its duration and scope (De Castro [2004] p.104; [2001] in clickable bib.).

In the slave plantation, the workers did not have the right to quit. In the capitalist sugar mill, they did. Obviously both plantation and mill are inheritable. At abolition, the capital value of the plantation would be seriously compromised because the workers would gain the right. Only if under slavery this right had no value to the slave would a devaluation of the capital stock not occur. But slavery in this case would be redundant. In the jargon of contract theory, the incentives would have satisfied the slave's Participation Constraint so that even if slavery were to be abolished, the now free worker should still remain on the job, as some did.

Piketty calls $\alpha_K = rK/Y$ the First Fundamental Law of Capitalism but admits immediately that it "*is a pure accounting identity*" (Chap.1 p.52). He justifies the term in that "*it expresses a simple, transparent relationship among the three most important concepts for analyzing the capitalist system*". We agree with this interpretation but quibble about the degree. Technical progress, in our view, is even more central to what happened since modern economic growth began, the main historical phase covered by his research.

Since his data show that (pre-tax) r has varied little, almost always between about 4.5% and 5.2%, for Britain since 1770 (Fig.6.3 p.202), and France since 1820 (Fig. 6.4), his attention turns to the possible long-term trajectories for β ($\equiv K/Y$ not $\Delta K/\Delta Y$) in determining $\alpha_K = r\beta$, the share of income accruing to wealth. For this he introduces his Second Fundamental Law, $g = s/\beta$, in its version $\beta = s/g$, to substitute out β in the First Law to yield $\alpha_K = rs/g$, the equation he needs for his Prediction. The first version, he attributes correctly (Chap. 6 p.231) to Domar (see Point 3 below). The second version, $\beta = s/g$, he says was made possible by Solow's $F(K,L)$. We show now it was developed mainly by Trevor Swan [1956].

In our Point 2 below, we prove that $g = s/\beta$ is an identity, true from the national accounts at any time period t , only if β is defined as the *incremental* capital-output ratio $\Delta K/\Delta Y$. In the steady state phase of some of the theoretical models we treat here, $\beta \equiv K/Y$ is constant and so $\Delta K/\Delta Y \approx K/Y$ (see Points 4 and 5). However, Piketty does not make this assumption, not only because his data show large variations in β in the 20th century, but also because understanding the long-term trend in these variations is crucial for his Prediction. For this, he uses the Second Law in the version $\beta = s/g$ to claim that this value of β "*is the result of a dynamic process: it represents a state of equilibrium toward which an economy will tend if the saving rate is s and the growth rate g , but that equilibrium rate is never perfectly realized in practice*" (Chap.5 p.169).

We show now that this is a result from Trevor Swan's [1956] study of a different definition of steady states (SS) from the one used by Solow. For a Swan SS, $\beta \equiv K/Y$ must be constant and not k ($\equiv K/L$), as in standard Solow. With technical progress in Solow however, k can continue to grow in a Swan SS. In fact, the growth literature has come to refer to this as one attribute of "balanced growth". Swan [1970, original 1960] named paths with constant β , Golden Ages, as Joan Robinson called them (see also Appendix I here).

The log differentiation with respect to time of $\beta \equiv K/Y$, yields $gr(\beta) = gr(K) - gr(Y)$, where the notation $gr(x)$ is the exponential growth rate of x . From the Keynes multiplier $I = sY$, and given $I \equiv dK/dt$, we obtain $gr(K) = sY/K = s/\beta$. For any constant $gr(Y) = g$, therefore, the condition for β to be both constant and globally stable is $s/\beta = g$. That is, for an economy with constant g and s , β will tend to the value s/g . Trevor Swan [1956] studied the dynamics for β in the Solow model. For the standard version without technical progress, both *SS* conditions hold. The Solow *SS* yields $gr(Y) = gr(L)$ so that $gr(\beta) = 0$ also when $gr(Y) = n$, the exogenous growth rate of the labor force, $gr(L)$. For growthmen, however, this is a stationary state of no growth since income per worker, Y/L , is stagnant. The UN projections for g adopted by Piketty explicitly envisage positive growth for Y/L in the 21st century (Fig.2.4 p.100). So the standard Solow model without technical progress is not consistent with the assumptions he used for the Prediction. Further, since Piketty's data found the (pre-tax) rate of return, r , to be quite stable, we will show in Point 7 that with Harrod-neutral technical progress, the Swan *SS* will reproduce this observation and yield both long-run $g = g_A + n$, and $gr(\beta) = 0$ when $\beta = s/g = s/(g_A + n)$, where g_A is a constant, exogenous rate of technical progress.

There is a further fit of this Swan *SS* result, $g = g_A + n$, to the UN projection of g he uses for the Prediction. Given the fall in g for 2050-2100, from 3.2% to 1.6% (Fig 10.9 p.354), about a third of the fall will be due to lower population and hence lower labor force growth, n , from 0.7% in 2012-2050 to 0.2% per year (Table 2.3 p.79). This leaves the possibility that our constant g_A assumption may remain valid while the projected g falls. With s held constant, lower long-run g implies higher long-run β , which tends to s/g , Piketty's Second Law. Thus we can use technical progress to replicate his Prediction of an exploding trajectory for $\alpha_K = rs/g$ without the need to assume $r > g$. This makes clear that Piketty's assumption, $r > g$ almost always, is required, not for his Prediction, but for his *Central Contradiction of Capitalism*, his prognosis of exploding inequality of inherited wealth in the 21st century (Conclusion, last Chapter, not numbered, pp.571-3).

Debraj Ray [2014], another reviewer, proves $r > g$ to be always true in a "*Harrod-style growth model*" (spuriously). In Appendix I here, we explain the theoretical flaw in Ray's proof. This explanation then enables us, in Appendix II, to prove that both $r > g$ and $g > r$ are theoretically possible in the two cases, Harrod-Domar and the Swan *SS* in the Solow model with Harrod-neutral technical progress. For the Swan *SS* in this non-standard Solow, α_K is always constant, independent of whether the elasticity of substitution between K and L , $\sigma = 1.0$ or not.

As we pointed out earlier, both Piketty and Acemoglu & Robinson ([2014] Appendix) studied an inheritance theory which can determine the changing parameters over time for the fat tails of a Pareto distribution for the richest wealth holders. Piketty obtained a result in which a $(r - g)$ gap of (5% - 1%) can give rise to a long-run equilibrium distribution in which the wealth

concentration is such that around 90% of the capital stock are owned by the top 10% , and over 50% by the top 1% of the holders of wealth (Piketty [2014] Chap.10 pp.364-5).

The technical problem with this focus on the long-run equilibrium is that such distributions are not unambiguous indicators of the stochastic processes which generate them. For example, two very different processes can generate the same type of distribution, as the study by De Castro & Gonçalves [2001, 2003] illustrated. Solow [1997] had used Monte Carlo simulations in a model with a lock-in stochastic process to reproduce the emerging bimodal world distribution in the post-war *GDP* per capita data. The model started from a set of economies with identical initial incomes per capita and, in each period, the probability of an economy moving up or not in the ranking depended on its evolving, past accumulated successes. In the model's long-run bimodal distribution, the economies left at the bottom would be locked into their poverty because the probability of a move-up in the distribution would have become extremely small. There would be little "churning" in this bimodality. The churning in a distribution is the extent to which items change their rank in it e.g. the % of the rich no longer rich after a given interval. This property would seem to be as relevant to public policy as the long-run wealth shares, but Piketty does not study it.

In De Castro & Gonçalves the probabilities that each economy move up or not were always drawn at random from the same endogenous probability density function (*pdf*). For some parameter values of this *pdf*, the model reproduced the same long-run bimodality, that is, with no use of a lock-in type of stochastic process.

An empirical study of churning in the US labor income distribution is Burkhauser, Holtz-Eakin & Rhody [1997]. After dividing the distribution into quintiles, they estimated the % of US workers who changed their quintile in each of two decades, 1970s and 1980s. The least mobility was in 1st and 5th quintiles, the poorest and the richest. The highest was in the middle, the 2nd, 3rd and 4th where over 50% of each quintile moved either up or down within a 5-year interval. These are incomes from the labor market. Non-labor incomes are much harder to come by. As for inherited wealth, Piketty's special interest, he claims that, unlike France or Britain, the US data are unreliable since "*it is unfortunately impossible to compare the survey data with fiscal records*" (Piketty [2014] Chap.11 pp.427).

Nearly constant (pre-tax) r since "*antiquity*" (year zero BC, AD, CE) , discovered by Piketty for the world economy (Fig. 10.9 p.354), may well replace Kaldor's more historically restricted one, α_K constant since modern growth began. However, more or less constant r may be a further consequence of the Modigliani-Miller theorem that the rate of return on capital does not depend on the method of its financing. The valuation, K , given to the physical and other assets in the enterprise would adjust in the capital market to standardize its rate of return, r , driven by some universal behavioral trait such as a more or less common, primordial, myopic discount rate for future consumption. This may be a version of A. K. Sen's assumption of

constant r studied in Point 5 here. Sen claimed it was determined by competition in the capital market, at the minimum return to induce entrepreneurs to invest. Certainly, Piketty's new discovery should help to remove from growth theory the conjecture that cultural traits generated differential discounting of future consumption and thus may have been a probable cause of the Great Divergence.

In Point 7, we show that in Solow-Swan with a constant saving rate, Harrod-neutral technical progress can make α_K vary either way on the transient trajectory, even as the economy moves to a steady state with constant α_K . Point 6 treats with other types of technical progress. Both Hicks- and Solow-neutral will yield no viable Solow-Swan SS since α_K will go to zero or 1.0 in the long run, except when the elasticity of substitution in Solow's $F(K,L)$, $\sigma = 1$.

Also in Point 7, we show how fiscal policy can be used to vary the saving rate, s , to keep α_K constant, even when $\sigma \neq 1$, whether or not a Solow SS exists. Further, this policy will track the rate of technical progress, the true engine of growth. This conclusion is the consensus of the recent literature on endogenous growth which moved away from the earlier emphasis on the accumulation of physical capital.

Numerics with long-run α_K :

Piketty's data [2014] show (Fig.6.5 p.222) that for his 8 rich economies in 1970, α_K varied between 15% and 25% of national income, while in 2000-2010, it was between 25% and 30%. Neoclassical growthmen now use $\alpha_K \approx 30\%$ as the current Kaldor stylized fact for the capital share of most economies. With r held constant at 5%, $\alpha_K = r\beta$ yields a long-run β value of 6. Using world g at about 3% say, Piketty's Second Law, $g = s/\beta$, would give us a saving rate $s = 18\%$. If as he claims, world g can fall to 1.5%, then, with s held constant, the capital share will double to $\alpha_K = 60\%$, as will the capital-output ratio, β , to 12. On the other hand, China currently has both s and g at extremely high levels, let's say at about 42% and 7% respectively in round numbers, and let's use them to set current β roughly, say 7, and thus $\alpha_K \approx 35\%$. If China's g then halves in the later 21st century with r and s constant, his predicted α_K will go to 70%. It is true that some economists believe that Brazil's α_K is already there. We show now why they are wrong.

Brazil's α_K :

Many economists in Brazil have claimed that α_K here is as high as 60 to 70% (e.g. Bresser Pereira [2002], Table 1, p.125). The extremely high Gini indices of the personal (income) distribution in the post-war economy gave some credibility to these estimates. However, Victor Gomes [2002] in his PhD thesis for Eco-UnB, adopted several more careful methods and obtained different average α_K values for 1990-98 ranging from 33 to 50% (Table 4.2 p.141). The reason for the much higher estimates is that the federal statistical agency IBGE publishes 3 types of incomes, wages and profits of incorporated enterprises, and total income of unincorporated ones (called there "proprietors") such as small business and the self-employed

e.g. taxi drivers, lawyers. Most non-specialized researchers in Brazil usually just treat the third type as all profit when in fact most of it is labor income. Victor Gomes corrects this error.

For Brazil, 1950-2010, β ($\equiv K/Y$) has varied in both directions. The measured personal distribution, the Gini index, has been falling in the last two decades. However, this is no indication that the α_K is falling, firstly because the data used for the Gini, the PNAD surveys, are dominated by labor incomes. And secondly, it is also because capital incomes should appear in the upper 1% and 10% of the personal distribution where the data are weakest.

Point 2 Explaining $g = s/\beta$ using only national income accounting for a closed economy.

Proof: $s \equiv S/Y$ and so $S = sY$; $I \equiv \Delta K$; ex post accounting says $I = S$; $\beta \equiv \Delta K/\Delta Y$, so that $\Delta Y = S/\beta$. Now $g \equiv \Delta Y/Y = (S/\beta)/Y$ so that $g = s/\beta$. End of Proof.

Remarks: (i) The national accounts must satisfy this relation at every time period t . Note, however, that β here is not estimated as K/Y and so at any period t , observed β may not be equal to observed K/Y .

(ii) As we saw above, Piketty interprets $g = s/\beta$, with $\beta \equiv K/Y = s/g$, as the long-run equilibrium value to which β tends, given an economy with s and g held constant. This equilibrium is a Swan SS.

Point 3 Explaining $g = s/\beta$ as the required (“warranted”) growth rate for continuous full utilization of K ; this is the first of two main Harrod-Domar results. We have now to distinguish between Y , the observed GDP , and Y^* , the GDP if there is full K utilization. Using the Keynes multiplier $Y = (1/s)I$, and defining $g \equiv \Delta Y/Y$, we obtain $\Delta Y = (g/s)I$. Harrod-Domar introduced the accelerator equation, $\Delta Y^* = (1/\beta)I$, where β is now interpreted as a fixed, exogenous parameter determined by the aggregate technology of the economy when there is full utilization of K . The first Harrod-Domar result is that, for $Y = Y^*$ and $\Delta Y = \Delta Y^*$, the warranted growth rate is $g = s/\beta$, and $\beta = \Delta K^*/\Delta Y^*$ constant. Without these conditions, however, $\beta \equiv \Delta K^*/\Delta Y^*$ cannot be estimated by $\Delta K/\Delta Y$ from the national accounts.

Remarks: (i) The second Harrod-Domar result is that if at any period t , $g \neq s/\beta$, then the economy would never return to $g = s/\beta$, Harrod-Domar’s knife-edge instability for this value of g . When Solow introduced his $Y = F(K, L)$, he intended that the substitution between K and L would remove this instability. A.K. Sen [1970, original 1965] proved he was wrong. In both models, Harrod-Domar and Solow-Swan, the theory of expectations formation, by entrepreneurs when they determine I , is flawed because in it their chronic errors in expectations should be obvious to them. See Point 5.

(ii) Since Piketty claims that $g = s/\beta$ holds only in the long run as $\beta \rightarrow s/g$, and since he finds that β varied enormously since 1910 in the rich economies he studied, he should specify the complete dynamics. Solow, unlike Harrod-Domar, allows this long-run specification (see Points 4 and 6).

Point 4 $g = n$, the labor force growth; the steady state (SS) in Solow without technical progress. As we indicated above, Solow's SS becomes a stationary state of no growth in per worker income, Y/L . Solow also assumed implicitly that $Y=Y^*\equiv F(K,L)$ always. That is, investment I is always correct for full utilization of both K and L (no Keynesian recessions allowed).

Remark: For the longer run of growth theory, recessions, like booms, are largely irrelevant, as Lucas discovered after dedicating most of his early work to business cycles. China and India are poor today not because they had more recessions than the US or France.

Point 5 Explaining $g = s/\beta$ as the "warranted" growth rate even with Solow's continuous full utilization of both K and L , as demonstrated by A.K. Sen [1970; original 1965]. That is, the s/β growth rate becomes "warranted" with Solow's $Y = Y^* = AK^\alpha L^{1-\alpha}$ in place of Harrod-Domar's $\Delta Y^* = (I/\beta)\Delta K$. Conclusion: Solow's neoclassical Surrogate function, which allows substitution between K and L , did not remove Harrod-Domar's knife-edge instability. Sen used the Cobb-Douglas special case of Solow's $F(K,L)$ which cannot separate the different types of technical progress. See Point 6 which gives the more general case.

When entrepreneurs choose investment level I correctly for $Y = Y^*$, Sen added two assumptions. Firstly, that their expectations for r are always fulfilled at K 's marginal product, and secondly, that r is assumed to be constant, fixed by the competitive K market at the minimum required for them to make investments.

The first yields $r = \alpha Y/K$, $0 < \alpha < 1.0$, written as $\beta \equiv K/Y = \alpha/r$. Here α is the exogenous capital share, α_K , now fixed by the Cobb-Douglas parameter in Solow's aggregate technology and the two competitive factor markets for K , L . Reinterpreting the warranted growth rate equation as $\beta = s/g$, and substituting α_K/r for β we get once again the result $\alpha_K = rs/g$ that Piketty needs for his prediction.

However, the long-term dynamics for α_K , r and g can now be clearly specified. Sen's second assumption, that the K market keeps r constant as k ($\equiv K/L$) increases, requires Harrod-neutral technical progress where β and thus α_K must remain constant, if Y is to grow at its "warranted" rate, $g=s/\beta$. This g is now also determined by the steady state growth rate of Y/L , which is no longer zero but given by the sum of the rate of exogenous technical progress, the A in $AK^\alpha L^{1-\alpha}$, and n , the labor force growth rate. That is, $g = gr(A) + n$. Both of these rates are exogenous here. See Point 6.

. Thus Piketty's equation, $\alpha_K = rs/g$, would now also fix a required (warranted?) saving rate, s , the only parameter still free.

Remarks: (i) As Sen-Solow's assumptions fit most of the long-term stylized facts since modern growth began, Piketty's Prediction of increasing α_K in the 21st century would require non-warranted growth, in this version of Solow's neoclassical metaphor.

(ii) To see this, the saving rate s , the only remaining parameter in his equation, is now already fixed by α_K , r and g for “warranted” growth. Thus, only non-warranted growth would allow for independent behavioral saving theories (see Krusell & Smith [2014] in clickable bibliography).

Point 6 $g > n$, in the non-standard Solow model, that is, with exogenous, disembodied, technical progress at constant rate, g_A . We give 3 versions of Solow’s aggregate function. Only the first and last yield $gr(Y/L) = g_A$ on a Swan SS growth path. The second does not have a viable Swan SS. It is an example of Hicks-neutral technical progress (see below).

$$1. Y = BK^\alpha L^{1-\alpha} = K^\alpha (AL)^{1-\alpha} \text{ where } B \equiv A^{1-\alpha}.$$

Remark: $\alpha_K = \alpha$, constant for all paths. The Cobb-Douglas form cannot distinguish the 3 types of technical progress in the literature.

$$2. Y = AF(K,L), F(\cdot) \text{ crs in } K,L.$$

Remark: α_K here will vary along the paths, factors will receive their marginal products but a long-run Swan SS will not be viable because α_K will tend to either zero or infinity (see later in this Point 6).

$$3. Y = F(K,AL), F(\cdot) \text{ crs in } K,AL.$$

Remark: α_K here will vary but in the long run, α_K will be constant (see Point 7). This yields a viable Swan SS.

Proofs: Most elementary growth texts give these. At the graduate level, see B&D Chap.3.

In our Point 1, we claimed that only non-standard Solow fits Piketty’s Prediction. Here is why:

Result: $gr(\beta) = 0$ when $\beta = s/(g_A + n)$. That is, Piketty’s g in his Second Law, $\beta = s/g$, becomes $g = g_A + n$.

Proof: We have $gr(Y/L) = gr(Y) - gr(L) = g - n$, and in a viable Swan SS, $gr(Y/L) = g_A$.

So that, $gr(Y) \equiv g = g_A + n$ and $gr(\beta) = gr(K) - gr(Y) = s/\beta - (g_A + n)$.

It follows that when $gr(\beta) = 0$, we will have $\beta = s/(g_A + n)$. End of proof.

Remarks: (i) In the Cobb-Douglas case, $\alpha_K = \alpha$, constant over the entire trajectory, not just the SS, which would not fit Piketty’s 20th century data. In the general case, $F(BK,AL)$ with constant returns to scale (crs) in BK, AL , only Harrod-neutral technical progress ($B = I$) will yield viable Swan SS trajectories for α_K which do not go either to *zero* or to *1.0*. See Point 7.

(ii) That this exogenous g_A determines the long-run growth rate of Y/L is a fatal flaw in Solow-Swan growth theory since it can explain the “Great Divergence” since 1820-50 of *GDPpc* across economies only by differences in technical progress which is exogenous here. To overcome this was the research agenda of the new theories of endogenous growth. See Point 8.

The neutrality of any type of technical progress is about whether it keeps α_K constant. The types emerge from the study in (y,k) space of selected paths when technical progress shifts upward the per worker version of the aggregate production function but leaves the income distribution between K and L , α_K , unchanged. The three main ones in the literature are:

(i) Hicks-neutral: α_K constant for paths with k constant.

(ii) Harrod-neutral: α_K constant for $\beta \equiv K/Y = k/y$ constant.

(iii) Solow-neutral: α_K constant for $l \equiv L/Y$ constant.

Despite this terminology, we should point out that Solow [1956, Section VI] worked with the type which later came to be called Hicks-neutral. For this case, $A = B$ in $F(BK, AL) = AF(K, L)$ since $F(\cdot)$ is *crs*. The A term became known as the *Total Factor Productivity* of the aggregate technology, a concept much used in real business cycle macroeconomics.

Here we show that Hicks-neutrality cannot yield a viable long-run trajectory with β constant, a Swan *SS* balanced growth path. The trajectory for $gr(\alpha_K)$ is given by the following result:

Result: $gr(\alpha_K) = (1 - 1/\sigma)gr(A)$ where σ is the elasticity of substitution between K and L .

Proof: See Burmeister & Dobell (B&D) [1970] p.80 equation (15), pp.136-138, Chapter 5.3, and my lecture notes for Macroeconomics 2, the doctoral course at Eco-UnB. The latter is needed because B&D do not complete the formal proof. Instead they offer only hints. Maybe Acemoglu's textbook [2009] has one. End.

Remarks: (i) If $\sigma > 1$, $\sigma = 1$, or $\sigma < 1$, then α_K will respectively either increase, remain constant or decrease in the long-run trajectory with β constant. Thus, for $\sigma \neq 1.0$, Hicks-neutrality will not yield a viable Swan *SS* because α_K will go to either zero or 1.0 . In the Cobb-Douglas case, $\sigma = 1.0$ and α_K will always remain constant, even on the transient paths.

(ii) A similar result is available for Solow-neutral technical progress, where $Y = F(BK, L)$.

(iii) Piketty thinks that the "development process" may cause $\sigma > 1.0$ (Chap.6 pp.216-224). The data so far do not confirm this but Samuel Pessoa [2014] suggests that in the second Globalization, from about 1990 on, $\sigma > 1.0$ may now emerge due, not to increased aggregate technological flexibility, but rather to the new trade flows between the rich and poor economies, an open economy argument studied neither here nor by Piketty.

Point 7 With Harrod-neutral technical progress, $Y = F(K, AL)$. If $\sigma \neq 1.0$, then α_K can increase or decrease on the transient path but not in the steady state (*SS*). On the other hand, we show now that α_K can remain constant during both growth phases even with $\sigma \neq 1.0$, provided $gr(A) = gr(k)$, $k \equiv K/L$. This is our main counter-argument to Piketty's fear of the long-run α_K exploding in 21st century capitalism after 2050 when world g is projected to fall by half. Even in the transient phase, the saving rate s can be varied by fiscal policy to adjust $gr(k)$ to near equality with the more ephemeral $gr(A)$ and so keep α_K on the straight and narrow. With this policy, the engine of growth becomes technical progress ($gr(A) > 0$) and not the concomitant capital deepening ($gr(k) > 0$). Moreover, the assumption of Harrod-neutral technical progress frees the discussion from whether the elasticity of substitution, $\sigma = 1.0$ or not (see Samuel Pessoa [2014], appendix). Further, since capital always receives its marginal product here, this policy does not require that $r > g$, since α_K remains constant, independently of the relative values of r and g (see Appendix II here).

The relevant result for Harrod-neutral technical progress is the following equation, valid for the entire trajectory, not just the Swan SS:

Result: $gr(\alpha_K) = ((1/\sigma) - 1) (1 - \alpha_K) [gr(A) - gr(k)]$; where $\alpha_K \equiv rK/Y = r\beta$.

Proof (hints for): B&D does not give this equation, much less a formal proof. But it can be put together from the same sections of their book as in Point 6. The proof needs several steps but is not difficult. I've done it in my lecture notes for the same Macroeconomics 2 course cited there.

The intuition for the result comes from the definitions of $\alpha_K \equiv rK/Y$ and Harrod-neutrality which requires α_K to be constant for K/Y constant, the Swan SS. We can do this in two steps. The first must show that with K/Y constant, the marginal product of capital is constant. So if the rate of return to capital, r , is equal to its marginal product, then α_K must be constant. The second is that on the Swan SS, Harrod-neutrality must have $gr(\alpha_K) = 0$ and so $gr(A) = gr(k)$, except if $\sigma = 1.0$.

The capital market condition is $r = f_k(k;t)$ where f_k is the marginal product of capital, given the technology at time t , from $Y = F(K, L; t)$, crs in K, L to get $Y/L \equiv y = f(k; t)$. Harrod-neutrality requires paths with α_K constant along which $\beta \equiv K/Y = k/y$ is constant. Since $\alpha_K \equiv rK/Y = f_k k/y$ must be constant, this means all three terms, k/y , α_K and f_k are not independent. If any two are constant, then the third must be constant. Thus a shift in $y = f(k; t)$ is Harrod-neutral if and only if f_k is constant along a ray from the origin in (y, k) space with a constant slope y/k .

Finally, since the Solow model with Harrod-neutral technical progress must have a Swan SS with $gr(\alpha_K) = 0$, then from the equation, either $\sigma = 1.0$ or $gr(A) = gr(k)$. In the latter case, although k is increasing ($gr(k) > 0$), $r = f'(k)$ does not fall because technical progress ($gr(A) > 0$) compensates for the drop in r caused by the capital deepening ($gr(k) > 0$). Here, market forces keep α_K constant even when $\sigma \neq 1.0$. The intuition is that even with r constant, the wage rate, w , increases because of the more capital intensive techniques and thus increased labor productivity. That is, the factor price ratio, w/r , will increase continuously to adjust the factor-markets equilibria to the increasing k ($\equiv K/L$) to maintain a competitive equilibrium in both factor markets. But K/Y will remain constant on the Swan SS path. The only exogenous parameter is the growth rate of technical progress, $gr(A) = g_A$.

On the transient paths, only if the elasticity of substitution $\sigma = 1.0$, will constant α_K be preserved.

End of (intuitive) Proof.

Remark: Dani Rodrik has used this equation in his comments on, and published with, a paper by Susan Collins & Barry Bosworth [1996], equation B1, p.196. The source he cites is work by Diamond, McFadden & Rodriguez in Fuss & McFadden (Eds) [1978]. We've not yet managed to get access to this reference. Rodrik follows the literature to claim that, since the observed data $gr(\alpha_K) = 0$ can be caused by either $\sigma = 1.0$ or $gr(A) = gr(k)$, the econometrics cannot identify

which. We doubt this claim. In any case, in Piketty's 20th century data, $gr(\alpha_K) \neq 0$ which result negates one of Kaldor's main stylized facts about modern economic growth.

For the case $\sigma \neq 1.0$, a few further details on the economic properties of this $gr(\alpha_K)$ equation may help the exposition:

(i) If $\sigma \neq 1.0$ then $gr(\alpha_K) = 0$ only if $gr_A = gr(k) = gr(K) - gr(L) = s/\beta - n$ or $\beta = s/(g_A + n)$.

In the Cobb-Douglas case, $\sigma = 1.0$ so that $gr(\alpha_K) = 0$ even when this value of β , fixed by g_A , n and s in Solow, does not occur. With long-run β constant, the assumption of Harrod-neutral technical progress will ensure that r remains constant, and thus so will $\alpha_K = r\beta$ on this trajectory, the Swan SS in this Solow model. However α_K here will no longer be fixed by the Cobb-Douglas technology, nor will α_K be fixed through r held constant by the capital market, as Sen argued in Point 5.

(ii) If $\sigma < 1.0$, then $((1/\sigma) - 1) > 0$. For $g_A > ((s/\beta) - n)$, α_K will increase; β will decrease but r will increase with technical progress sufficiently to allow increased α_K . On the other hand, $g_A = (s/\beta - n)$ will ensure constant α_K .

(iii) If $\sigma > 1.0$, then $((1/\sigma) - 1) < 0$. For $g_A > (s/\beta - n)$, α_K will now decrease; β will decrease as in (ii) but r may now increase or decrease since α_K is now decreasing.

Point 8 When Lucas, Paul Romer, Matsuyama and many others introduced endogenous technical change and human capital in the late 1980ties, the plot thickened.

All these results are for a closed economy. Now the second Globalization opens it.

Point 9 After the collapse of the Bretton Woods regime from around 1970-80, international financial flows are freed and concomitantly, also exports of non-primary goods from poor to rich economies. Both flows explode. The very poor in the poor economy can benefit (earn more than US\$2/day) but both the *Gini* and α_K can explode in both types of economies, poor and rich (for theory, Kumhof and Rancière [2010], Matsuyama [2004]). The purported cause: financial deregulation and improved intermediation in the rich economy (read US), and primitive financial intermediation and huge current account surpluses ($X \geq M$ implies $S \geq I$) in the poor economy (read China) which are channeled through the rich economy (read Wall St) for subsequent real investment anywhere (not only US). The purported result: worker productivity and thus the median wage in the rich economy stagnate or fall there. In the poor economy, its poorer agents cannot borrow due to the primitive intermediation and its rich agents there send their surplus abroad, using the rich economy's superior financial intermediation (read Wall St), to be invested wherever. Sovereign institutions (e.g. central banks) also send some of their "savings" to the US (e.g. US\$ reserves). In this situation, α_K explodes in both types of economy, the rich and the poor.

These more recent, pessimistic theories seem to capture only the capital market effects of globalization. Earlier, Krugman and Venables [1995] studied its labor market effects, in particular the sequence of an initial divergence of the real wage across economies caused by the

early industrializers like Britain, France and the US, and a subsequent projected future re-convergence (our term) to a common value (see especially Fig.V p.871). This more optimistic prognosis, for what they dubbed the third stage, is based on falling trade costs, especially transport. Non-financial firms, they claim, become “*indifferent between locating in the core and the periphery*”. The real wage in the core may even fall slightly, though this result is not inevitable (see Fig.VI). In any case, the final result is complete wage-rate re-convergence. Krugman and Venables did not give a date for this but Piketty cited the UN projections which claimed that by 2050, absolute convergence of *GDP* per capita will have occurred (Fig.2.4 p.100). And since Piketty found (pre-tax) r to be nearly always constant, then *GDP* per capita convergence implies wage-rate convergence.

END of Points 1 to 9

Final remarks

Economic history analogy with the first Globalization: Some of this happened to Britain by about 1875 when the City had become a major conduit for *other folks'* finance capital. The US economic historian Leland H. Jenks, who studied the British case years ago, claimed that this fostered “the growth of a *rentier* governing class, whose interests lay outside the community in which they lived and exerted influence” (Jenks [1963, orig. 1927] p.334). Some critics accused the stock brokers in the City of starving British industry while they gave priority to investment opportunities overseas. Others defended them saying the flow was a two-way street, a world market for finance capital. In fact, we now know that in Britain’s economy then, neither *GDP* per capita nor real wages stopped growing, nor did its economic structure change much after. What changed was its politics, the emergence of the electoral power of the mass of its population, the working classes. The US political system already has this mechanism in place which should allow it, we think, to resolve the political tensions the current global transformation has generated in the rich economies – huge profits for the bankers, stagnant wages and mass unemployment for the workers.

As for outcomes, the Great Re-Convergence of per capita incomes across nations has already begun, led in this initial phase by mega-economies like China, India and a few others even earlier e.g. Brazil, Japan after 1950. Piketty suggested that the capital income share α_K in the world economy may explode as its *GDP* growth rate, g , falls by half after 2050 (Fig. 2.4 p.100). This will be more a distribution disaster than a growth one because he accepts the UN’s purported Great Re-Convergence by 2050. This predicted explosion of α_K is a replication of tendencies he found in the current rich economies before 1914. Rogoff’s conjecture [2014] is that the rich agents in the rich economies may be getting richer precisely because they are investing in the poor economies and so are currently lifting these out of their mass poverty. At least one stylized fact seems to suggest that outcome.

However, another current stylized fact seems to suggest that an alternative route to the Great Re-Convergence is through secular stagnation of the rich economies – the zero-growth syndrome of both *GDP* and population already experienced by some e.g. Japan and in most of the European Union (Piketty [2014] Chap.1, esp. 64-71). Certainly the rich economies do not have to become poor for the poor ones to become richer. They must however grow slower than the poor ones. Some of their rich agents may become even richer in the process. If this is due to increasing inherited wealth, they would be Piketty's *bêtes noires*. What he doesn't realize is that like the poor, the rich will always be with us. A few guys always get rich any way. Some rich may even stay rich. What is more important is growth. It is the poor who must not stay poor.

A third stylized fact, the reduced inequality Piketty uncovered in the rich economies for 1914-74, may well be repeated after 2050 in the re-globalized and re-converged world economy. In that first taming of unfettered capitalism, several new institutions were created – the progressive income tax, publicly funded higher education, state pensions, regulation of monopolies, not to mention world wars. For its second taming, Piketty gave us one suggestion, a common annual world tax on wealth (Chap.15 pp.515-530). In fact he believes that “*economic growth is quite simply incapable of satisfying [any] hope ...of a more just social order*” (Chap.2 p.96). Another suggestion, ours, is that the world should learn how to avoid having to use world wars as a redistributive mechanism.

Finally, unlike Piketty's emphasis on re-distribution, the policy recommendation for the current world economy which should emerge from the more recent growth literature is that technical progress in all its manifestations – education, R&D, transfer of know-how, paid for or otherwise, and trade in capital equipment, should be the mechanism, to a lesser or greater degree, for all economies, rich and poor. For the poor ones and therefore for the great majority of the world's population, the emphasis should continue to be on growth in *GDP* per capita, at least while there is such a huge gap with the rich economies. The current issue remains income distribution, not within nations except perhaps for the few rich economies, Piketty's contribution, but rather the one across them all.

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Appendix I: Nit-picking Debraj Ray’s proof of $r > g$ always, in Piketty’s $\alpha_K = rs/g$.

Ray’s proof (Debraj Ray [2014] pp.8-9) is set in a flawed version of a Harrod-type technology in which output Y is “linearly produced by capital so that the capital-output ratio is constant”.

In fact, the Harrod economy uses both K and L . However, Ray puts $Y = rK$, where r is both the return on capital and its constant output-capital ratio, Y/K . That is, he would have both $Y = rK + wL$ and $Y = rK$, a contradiction.

We correct now this contradiction by introducing the formal, aggregate production function implicit in Harrod's model, $Y \leq \text{Min} [vK, uL]$, with fixed coefficients, v, u . If the capital input K is assumed to be the limiting one always, and if the investment levels are always correct for full utilization of this capital stock, K (no Keynesian recessions), then Harrod's technology reduces to $Y = vK$. Thus $vK > rK$, that is, $v > r$ and NOT $r > g$ always. Here growth is "warranted" and $v \equiv Y/K = 1/\beta$.

This reasoning shows why one cannot use $Y = rK$ nor vK to obtain r or v as the marginal product of K . Harrod's implicit production function is not partially differentiable. To see this, even if we assume that K is always the binding input, any small increase in K requires a simultaneous complementary increase in L , before Y can increase. Further, even if we use Harrod's accelerator, $\Delta Y = vI$ and put $I \equiv \Delta K$, we still cannot claim that v is the marginal product of K , because even if K is binding at time t , Y_{t+1} still needs the complementary ΔL to obtain $Y_{t+1} = Y_t + \Delta Y$.

We present now Ray's flawed proof.

Debraj Ray's proof of $r > g$ always:

He rewrites the production function in his Harrod-style growth model, $Y = rK$, as $K/Y = 1/r$. In Piketty K/Y is β . From this he asserts that "*r is obviously the net rate of return on K and 1/r is the capital output ratio*".

He then uses Piketty's Second Fundamental Law, $g = s/\beta$, where $\beta \equiv K/Y = 1/r$ to get $s = g/r$.

Since the saving rate $s < 1.0$ always, then $r > g$ always.

End of proof.

Remarks: (i) Debraj Ray (p.7 bottom) assumes the mild *local* approximation $K/Y \approx \Delta K/\Delta Y$ at each period t . However, this extrapolates to the strong *global* assumption that K/Y is constant for all t , one which Piketty's 20th century data contradict. See Point 1 above.

(ii) On a historical note, Harrod did not think of his accelerator as a production function but rather, like Keynes, as a flow of funds condition. This was because he was introducing the dynamic effect of investment I in Keynes's static multiplier, namely on the increase in K it induces, $I \equiv \Delta K$, in the succeeding period. We are at the birth of a new and still poorly understood sub-discipline in macroeconomics. Harrod spoke only of I , not K , in his acceleration principle, $\Delta Y = vI$. This is why Joan Robinson was so upset when Solow introduced K in his aggregate production function, $F(K, L)$. And Joan was not a lady to mince her words. As she put it so vividly, Mr. Solow's K would seem to have the same plastic properties as shit since, even as techniques changed, any scalar quantity K could always represent all sorts of new and reconfigured old machines. Trevor Swan ([1956] p.344) was more polite – he saw Solow's K as Meccano sets. Solow's metaphor, it turned out, was inadequate for the analysis of business cycles (see Point 5 above), but it was powerful enough to launch a new branch of macroeconomics, growth theory.

Appendix II: No clear ordering $r > g$ or $g > r$ exists in Harrod-Domar and Solow.

Despite this indefiniteness, Point 7 above showed that in steady state (SS) Solow with Harrod-neutral technical progress, α_K is constant. This result contradicts Piketty's Prediction that when g falls, long-run α_K must explode, since here $g = gr(A)+n$, determined by two exogenous parameters which can be reduced and still preserve the SS α_K constant result.

Harrod-Domar does not yield a trajectory for α_K . In the following analysis, we keep the simplification that the saving rate $s \equiv S/Y$ is constant. In Harrod-Domar, "warranted" growth assumes that $\beta \equiv K/Y$ is an exogenous constant given by the aggregate technology (from $v \equiv 1/\beta$). However, no theoretically established value exists for r . In the SS, this Solow yields both β and r constant and endogenously determined by the assumptions of Harrod-neutral technical progress and equilibrium in the two factor markets, K and L . Equilibrium here means that these inputs receive their marginal products and are fully utilized.

In both models, constant β implies that Piketty's Second Law, $\beta = s/g$, goes through, though with different economic assumptions. In Solow, β will reach s/g only in the long-run SS, while in Harrod-Domar, warranted growth requires $\beta = s/g$ along the whole trajectory.

Debraj Ray's proof of $r > g$ in Harrod used $Y = rK$ and K/Y constant to obtain $g = rs$ which with $s < 1.0$ gave his spurious result. Here we will show that $g > rs$ in both Harrod-Domar and Solow. We show first that this condition does not yield an ordering for g relative to r .

When $s < 1.0$, $g > rs$ yields both possibilities, $r > g$ or $g > r$. To see this, if $s = 1/2$ say, then $g = 3r/4 > r/2$ which means $r > g$. But $g = 4r/3 > r/2$ also satisfies the inequality even though now $g > r$.

Result: $g > rs$ in Harrod-Domar.

Proof: From $Y = rK + wL$, we have $Y > rK$ from the national accounts.

But $Y = vK$ from $Y \leq \text{Min} [vK, uL]$ and assuming that K is always the limiting factor and therefore growth is "warranted".

In Piketty, $v \equiv Y/K = 1/\beta$ and from Harrod-Domar, $g = s/\beta$ in warranted growth.

Thus $Y > rK$ becomes $Y/K > r$. That is, $g/s > r$ or $g > rs$.

End of proof.

Remark: Harrod-Domar does not fix r at the marginal product of capital, K .

Result: $g > rs$ in Solow SS with Harrod-neutral technical progress.

Proof: $1/\beta \equiv Y/K > r$, the marginal product of K . In this Solow SS, both β and r are constants.

For constant s , g , Swan's SS will yield a stable, long-run value for $\beta = s/g$.

Substituting this in $Y/K > r$ we get $g/s > r$ or $g > rs$ for $s < 1.0$.

End of proof.

Remark: In this Solow SS, r is constant but w , the wage rate, increases continuously. However, α_K remains constant because as the factor price ratio w/r increases, the economy adjusts the K/L

ratio to use techniques more intensive of K , but in such a manner as to maintain a Swan SS , namely $\beta = K/Y$ constant. For details see Point 7.

These results do not contradict directly Piketty's Prediction that α_K can double if world GDP growth rate, g , falls by half. For this he used his Second Law, $\beta = s/g$, and then multiplied by a constant r to get $\alpha_K \equiv rK/Y = r\beta = rs/g$. We saw, however, that this value s/g is a stable Swan SS to which β tends in the long run only if s and g are held constant. Piketty's Prediction for long-run α_K is based on an UN projection that during 2050-2100, world g will fall by half (Fig 10.10 p.356; also Helsinki seminar, slide 17). This projection suggests that he does not envisage that between 2050 and 2100, g will remain constant at the 3.2% p.a. he accepts for 2012-2050, which is only slightly less than the 3.8% p.a. he found for 1950-2012. With g trending downward in the second half of the 21st century, therefore, the world economy cannot as yet be tending to a Swan SS . Thus long-run β cannot be identified by $\beta = s/g$, at least not much before 2100 and perhaps never.

Appendix III: $r > g$ implies increasing both inherited wealth and its concentration (Piketty's Central Contradiction of Capitalism)

Here we study first how Piketty outlines a theory for his empirical discovery that $r > g$ in the world economy since "*antiquity*" (Fig.10.9 p.354). Then we examine briefly the stochastic models he uses to link a possible future increase of inequality in the wealth distribution in the 21st century to an increase in the gap ($r - g$) (Piketty [2014] online Technical Appendix, p.60-2).

All the α_K results, Piketty's and ours, are based mainly on the Harrod-Domar and Solow-Swan models and their extensions. In Appendix II above, we showed that these yield no unambiguous ordering, neither $r > g$ nor $g > r$, even when α_K is constant as in the Swan steady-state of a non-standard Solow, namely with Harrod-neutral technical progress (see Point 7). This is a result in positive economics. In the normative economics of standard Solow, the introduction of criteria for the socially desirable saving rate, s , leads to Edmund Phelps' Golden Rule of capital accumulation: the saving rate s for constant maximum consumption per worker in the standard Solow SS must satisfy $r = n$, the growth rate of the labor force. In Solow-Swan, though not in Harrod-Domar, capital always receives its marginal product so any reference to r is also a reference to the slope of $f(k)$ in (y,k) space.

We show now how Piketty then uses the standard Solow SS result, $g = n$, and the Swan SS result, $K/Y = s/g$, to prove that if $r = g$, then $\alpha_K = s$, and that $\alpha_K > s$ if and only if $r > g$ (Piketty [2014] Chap.16 p.563, footnote 41 p.652; online Technical Appendix pp.82-4).

Result: If $r = g$, then $\alpha_K = s$.

Proof: $\alpha_K \equiv rK/Y$; $\beta \equiv K/Y = s/g$ in the Swan SS .

Thus $\alpha_K \equiv rs/g = s$ since $r = g$.

End of proof.

Remarks (i) Piketty's interpretation of this $\alpha_K = s$ result is:

"...in order for the golden rule to be satisfied, one has to have accumulated so much capital that capital no longer yields anything." (Chap.16 p.563, bottom).

(ii) For this interpretation, Piketty is assuming that only wealth holders save and invest, and that they do not want to work. So when all their income from capital has to be used to keep both K/Y and K/L constant, none is left for their consumption. The golden rule provides in the steady state, a maximal, constant level of consumption per worker in the economy.

Result: $\alpha_K > s$ if and only if $r > g$.

Proof: Once $K/Y = s/g$, this result is trivial.

Remarks: (i) By extension of his interpretation of $\alpha_K = s$, he argues that $\alpha_K > s$ implies that holders of capital will want to earn a larger share than s to allow them some consumption, without having to work. We quote him: "*The inequality $r > g$ is the basis of a society of rentiers*", Piketty's term for the idle rich (Chap.16 p.564).

(ii) The intuition for $r = n$ in Phelps Golden Rule comes from seeing that any SS with $r > n$ or $r < n$ will lead to a smaller constant maximum for consumption per worker. Put differently, such SS are dynamically inefficient (B & D Chap.2-7 p.52). For example, $r < n$ is dynamically inefficient because capital would be so plentiful that even a free gift of more of it would hurt more than help. The capital-labor ratio, k , is already so high that saving and sacrificing to maintain it would be yielding less steady state consumption for workers.

Despite these assumptions about the inter-temporal preferences of wealth holders, Piketty, however, does not claim that $r > g$ must always be observed. We quote him:

"To be clear, I take [$r > g$] to be a historical fact, not a logical necessity" (Chap.10 p.353).

Even so, his data show that $r > g$ has been an "*incontrovertible historical reality*" for the world economy since "*antiquity*" (Fig.10.9 p.354). But his data also show that from 1914 on, after-tax r , \underline{r} , became so much lower than r , and the GDP growth rate, g , increased so much that ever since, $g > \underline{r}$ holds (Fig.10.10 p.356). Nevertheless Piketty insists that in the 21st century, the world may return to the "*patrimonial capitalism*" which prevailed before 1914.

The theoretical basis for this prognosis by Piketty needs a separate paper. In the book, he cites the online Technical Appendix for the detailed equations (Chap.10 p.364, footnote 25). However, a further reference is given there to a Piketty & Zucman paper being prepared for the *Handbook of Income Inequality*, Volume 2, North Holland, 2014. It is this paper (93 pages) which has the mathematical details, and more interestingly, presents the results for the after-tax case ($\underline{r} - g$) in clearer perspective.

Here we provide a feel for the arguments, more to motivate other researchers (and ourselves, why not?) to examine further this important topic. We focus now only on how inherited wealth can cause inequality in the long-run equilibrium wealth distribution. Like with other authors, inheritance in Piketty depends on both demographics and β (Chap.11 p.383). In

the Swan SS, $\beta \equiv K/Y = s/g$. Once again, with no technical progress and for given s , if projected g falls after 2050, β will explode and so can the inherited share of total wealth. However, the inequality in the wealth distribution requires the extra argument on how the saving rate, s , is composed. For example, in the pure lifecycle theory, agents die leaving zero wealth and so the inheritance flow and hence the inheritance share are always zero, independently of the growth rate, g . Even so, to avoid a trivial degenerate, inherited wealth distribution, heterogeneity across agents must be introduced. One example is the assumption of exogenous random tastes for leaving bequests.

Finally, while the models use simulations to identify the shape of the long-run equilibrium distributions, usually a Pareto power law coefficient, Piketty does not study the churning which may occur along the growth trajectory. Equilibrium distributions are ambiguous indicators of the stochastic processes which drive them. For example, you can have two identical distributions of wealth across agents yet in one distribution families may be locked in to their disinheritance while, in the other, large numbers are no longer poorly endowed at birth after say one or two generations; ditto for the rich. In the second, there is no lock-in but rather what is dubbed significant “*churning*” in the distribution. This notion is tied up with measures of social mobility, persistence and even history dependence central to modern social democracies. Understanding these properties is crucial for public policy.

Persistence, for example, is related to Piketty’s notion of *inheritance* of wealth but it may not be identical. *Persistence* may be due more to merit e.g. good choices of investments (Piketty’s untested null hypothesis). *History dependence* is a test of whether the stochastic process generating the trajectories of family wealth has changed over the horizon of study. If it didn’t, we can say the family wealth distribution is *history dependent*. If the process changed along the way, then it is *path dependent*. In narrative economic history texts, these two concepts can be quite fuzzy and sometimes even used as synonyms in the sense that the *path* is the *history*. The time series for most individual families can be quite short and may not even span more than one generation. In such cases, the research may have to resort to profiling.

Piketty’s concern for the connection of inheritance to the worsening of the wealth distribution is seen most clearly in this quotation:

“When the rate of return on capital significantly exceeds the growth rate of the economy (as it did through much of history until the nineteenth century and as is likely to be the case again in the twenty-first century), then it logically follows that inherited wealth grows faster than output and income. People with inherited wealth need save only a portion of their income from capital to see that capital grow more quickly than the economy as a whole. Under such conditions, it is almost inevitable that inherited wealth will dominate wealth amassed from a lifetime’s labor by a wide margin, and the concentration of capital will attain extremely high levels – levels

potentially incompatible with the meritocratic values and principles of social justice fundamental to modern democratic societies” (first Chapter, Introduction, not numbered, p.26).

Inheritance inequality and not **income** inequality seems, at first sight, to be his fundamental objection to private ownership of capital or wealth. This is more John Stuart Mill than Karl Marx! Around the same mid-century period as Marx, Mill proposed a steeply progressive inheritance (“estate”) tax, on the grounds that the aristocracy, the landowners in what was still largely an agrarian economy, was a decadent class of idle rich who should be replaced by the emerging capitalists of the new industrial sectors. Consistent with his objection to inherited wealth, Mill was against a progressive tax on earned income since he saw it as “a burden” on the incentives of working people.

It turns out, however, that Piketty’s primary objection to 21st century capitalism is not the projected increasing inequality, whether inherited or earned, but rather that the increasing incomes from capital or wealth should allow their holders to enjoy a high standard of living without working – the return of the idle rich. We quote him:

“Rent is not an imperfection in the market: it is the consequence of a “pure and perfect” market for capital, as economists understand it: a capital market in which each owner of capital, including the least capable of heirs, can obtain the highest possible yield on the most diversified portfolio that can be assembled in the national or global economy. To be sure, there is something astonishing about the notion that capital yields rent, or income that the owner of capital obtains without working. There is something in this notion that is an affront to common sense and has in fact perturbed any number of civilizations, which have responded in various ways, not always benign, ranging from prohibition of usury to Soviet-style communism. Nevertheless rent is a reality in any market where capital is privately owned. The fact that landed capital became industrial and financial capital and real estate left this deeper reality unchanged.....To recapitulate: the fundamental force for divergence [in the personal income distribution], which I have emphasized throughout this book, can be summed up in the inequality $r > g$, which has nothing to do with market imperfections and will not disappear as markets become freer and more competitive” (Chap.11 pp.423-4).

This is certainly more Karl Marx than John Stuart Mill since it seems that the definitive solution Piketty seeks for the 21st century cannot be a straight replication of the taxation of capital incomes applied in the 20th. One possible interpretation of his position is that only the almost complete abolition of the private ownership of capital would avoid a drastic resurgence of the idle rich as g falls. However, if there is any lesson to be learnt from 20th century “*Soviet-style communism*” it is that the microeconomic distortions induced in the production structure of an economy by the complete removal of the incentives for private ownership of capital, eventually overwhelms the collective’s power to correct them. Certainly since 1989, the conventional wisdom would appear to be that despite the distortions that monopolistic

capitalism had induced in the previous 100 years, some governments were much more successful in moderating them sufficiently to provide a decent standard of living for the majority of their populations. Halfway through the process, say by about 1950, the others became identified as the so-called third world. For these, and perhaps even for the successful ones, the so-called first world, the issue remains moot. Bringing clarity to that debate may well be Piketty's major contribution.

End of paper