

Charter Schools, Equity and Efficiency in Public Education

**Reynaldo Fernandes
Naercio Menezes Filho**

**Julho, 2016
Working Paper 103**

Todos os direitos reservados.

É proibida a reprodução parcial ou integral do conteúdo deste documento por qualquer meio de distribuição, digital ou impresso, sem a expressa autorização do REAP ou de seu autor.

Charter Schools, Equity and Efficiency in Public Education

**Reynaldo Fernandes
Naercio Menezes Filho**

Reynaldo Fernandes
University of São Paulo

Naercio Menezes Filho
Insper and University of São Paulo

Charter Schools, Equity and Efficiency in Public Education¹

Reynaldo Fernandes

University of São Paulo

Naercio Menezes Filho

Inspere and University of São Paulo

Abstract

An increasing number of papers assess the effectiveness of charter schools by comparing the performance of their students with that of students in traditional public schools. In this paper, we develop a model that shows that the presence of charters reproduces a market equilibrium in which all students have the same purchasing power to buy educational services. In this situation, there is no difference in performance between the students allocated to charter or to traditional public schools. Nevertheless, the introduction of charter schools makes the system more equitable and, under certain assumptions, improves the average performance of all students.

Keywords: Charter Schools, Education Quality, School Choice

JEL Classification: I20, I22, J28

¹ We have benefitted from comments and suggestions from Eduardo Andrade, Miguel Foguel and seminar participants at the REAP workshop at Inspere and the Brazilian Econometric Society Conference. Emails: refernan@usp.br and naercioamf@insper.edu.br

1) Introduction

Charter schools are publicly-funded schools that, as public schools, cannot select students or charge tuition. They are exempt from certain regulations that affect traditional public schools and have more autonomy over decisions related to hiring and compensation, curriculum, school hours and teaching methods. Their defenders argue that they are more efficient and improve the performance of competing traditional public schools. Their critics claim that they draw resources away from traditional public schools and promote segregation. In the United States, approximately 1.8 million students attended a charter school in 2011, accounting for approximately 4% of all students in American public schools.²

This paper examines whether the complete removal of restrictions for the opening and expansion of charter schools would improve the quality of public education as a whole. This question goes beyond comparing the results of charter schools with those of traditional public schools. We argue that because traditional public systems face restrictions on optimally hiring and allocating teachers of different qualities among their students, charter schools may improve the quality of public education through a purely allocative mechanism. This happens even if charters are not better managed than the traditional public schools and even if public schools do not react to the loss of students caused by the presence of charter schools.

Several studies have examined the effectiveness of charter schools. In general, the aim of these studies is to compare the results of charter schools with those of the traditional public schools, somehow accounting for the students initial conditions. The results, however, are mixed. Some studies use longitudinal data to estimate models with student-level fixed effects,³ usually finding that the effect of charter schools on student performance is insignificant or negative (Clark et al., 2011). Other studies take advantage of the fact that, by having more candidates than vacancies, many charter schools admit

² A survey on the results and characteristics of charter schools is available in Epple, Romano, and Zimmer (2015).

³For example, see Sass (2006), Bifulco and Ladd (2006), Booker et al. (2007), Hanushek et al. (2007), and Zimmer et al. (2009).

their students based on a lottery. In this case, students who were not selected by the lottery serve as the control group of the experiment. Because participation in the treatment and control groups is random, such studies are the most statistically reliable. Studies using this approach generally find significant positive results in favor of charter schools.⁴ The problem with these studies is their external validity.

Studies of the first type (longitudinal) tend to have a broader scope, covering various educational districts or states, whereas the second type of studies (lotteries) focus on minority students and on large urban districts. For example, Clark et al. (2011) present the results of a randomized national study that includes 36 charter schools in 15 U.S. States finding that, on average, the performance of charter schools is no different from that of traditional public schools. However, the results vary significantly among schools and students, with positive effects for disadvantaged students and schools and negative effects for the most socially advantaged students and schools. Similarly, Angrist et al. (2013) find that, in the state of Massachusetts, the impact of charter schools is positive in urban areas (where the population tends to be socially disadvantaged) and negative (or not significant) in non-urban areas.

The fact is that charter schools substantially vary in design and the types of students they serve. State regulations vary widely among states, both in relation to the means of financing and to restrictions on opening and expanding charter schools. This diversity of situations can generate a variety of results. For example, if excess demand is an indicator of school quality, then studies that consider only charter schools with excess demand may be comparing the best charters with the average traditional public ones.

This article shows that the market mechanism underlying charter schools exerts a pressure to even the performance of students in different schools. If charter schools initially perform better, then more students from traditional public schools will want to transfer to them. Because the resources of charter schools depend on the number of enrolled students, these students are likely to obtain the desired vacancies. If the average performance of the school is reduced with the number of students, then there will be no difference, in equilibrium, between the performances of students in both types of schools.

We analyze the case in which the rules imposed on traditional public schools lead to an inefficient allocation of resources. In this scenario, the presence of charter schools can raise the average performance of all students in addition to producing equal treatment

⁴ See Hoxby and Rockoff (2005), Hoxby et al. (2009), Dobbie and Fryer (2011), Abdulkadiroglu et al. (2009), and Angrist et al. (2012).

for similar students. In this sense, charter schools restore the market conditions under which each student has the same amount of funds to buy educational services and schools are privately managed, but prevented from selecting their students. However, charter schools do not necessarily eliminate traditional public schools and in the end there will be no differences in students' performance, regardless of the type of school attended.

Our model is compatible with some stylized facts about charter schools. In addition to the issue of the performance difference between charter schools and traditional public schools, descriptive evidence shows that teachers in charters are on average younger, less experienced, and less qualified (degrees/certifications) and earn less than teachers in traditional public schools (Goldring, Gray, and Bitterman, 2013; Epple, Romano, and Zimmer, 2015). Moreover, Chudowsky and Ginsburg (2012) use data from the 2011 National Assessment of Educational Progress (NAEP) to compare the class sizes between charter schools and traditional public schools in major American cities, where most charter schools are located, and find that classes tend to be higher in traditional public schools (4th and 8th grade).⁵ These stylized facts are consistent with the model described in this paper, since one of the possible equilibria occurs with charter schools specializing in hiring teachers with lower teaching skills who teach smaller classes and earn a lower wage.

A related literature that has been growing in recent years examines the voucher system. Friedman (1962) was the first to argue that a voucher system would increase efficiency in the use of public resources in education. Epple, Romano, and Urquiola (2015) summarize the theoretical and empirical literature on the effects of vouchers on the quality and equity of the education system. It is important to emphasize the differences between vouchers and charter schools. In the first case, each student receives a voucher to buy educational services in private schools. If private schools are better managed, this mechanism may increase the efficiency of the system, depending on the conditions under which the voucher system is introduced (Epple, Romano, and Urquiola, 2015). Competition for students can also increase the efficiency of public schools.

If private schools cannot select students based on initial skills, cannot charge tuitions higher than the voucher value and if the voucher is available for all students, then

⁵ For example, 61% of 8th-grade mathematics teachers in traditional public schools report having more than 26 students in their classes, whereas this proportion is 37% for teachers of the same subject and grade in a charter school. Data from the NAEP show that, even when considering only large cities, teachers in charter schools tend to be less experienced and less qualified (degrees/certifications).

the voucher system becomes similar to the case of charter schools examined here. Under these conditions, the analysis developed in this article would also apply in the case of vouchers. Hence, we can regard charter schools as a special case of vouchers that seek to overcome the criticism that traditional voucher systems generate or reinforce stratification. In recent years, charter schools have been more common than voucher systems, perhaps because they are more politically palatable.

The structure of the paper is as follows. In the next section, we describe the general problem: an educational manager who must allocate his homogeneous students among teachers of different skills. In section 3, we examine the case of a central planner who has complete information and no restrictions. Section 4 presents the market allocation, in which each student receives the same amount to buy educational services, and section 5 presents the case of the manager operating in the constrained public system. Section 6 compares the resulting allocations in the different models, and section 7 concludes.

2) The Problem

Let us assume that the agent responsible for public education has a volume R of resources to provide education for N homogeneous students. This agent can choose between two organizational alternatives: the traditional public system and the “market” system. In the former, the government offers educational services directly, hires teachers and allocates them to students. In the latter, the manager divides R equally between students who buy educational services in the “market”. In the case of the traditional public system, the manager would be subject to certain rules that are discussed below.

There are two types of teachers: type 1 and type 2. The wages of teachers constitute the only cost of education. The quality of education is determined only by the quality of the teacher and the class size, that is, management quality is the same in all schools.⁶ Defining y_i and ϕ_i , respectively, as the performance and class size for each student under a type i teacher, we have the following:

⁶ The benefits of smaller classes have been highlighted in studies using experimental and quasi-experimental techniques (e.g., see Krueger, 1999; Angrist and Lavy, 1999; Schanzenbach, 2010). On the other hand, it has been widely accepted that the main determinant of the quality of schools is the quality of their teachers, although the effectiveness of the teacher cannot be easily associated with his or her observable characteristics (see Hanushek and Rivikin, 2006).

$$\begin{aligned}
y_1 &= f(\phi_1) \\
y_2 &= g(\phi_2)
\end{aligned}
\tag{1}$$

with $f'(\phi_1) < 0$ and $g'(\phi_2) < 0$. For any feasible class size ϕ^* , $f(\phi^*) > g(\phi^*)$, i.e., the type 1 teachers are more productive than the type 2 teachers. We also assume that the elasticity of performance in relation to the class size is lower than unity in absolute terms: $|\varepsilon_{\phi_i}| < 1$.⁷ Total educational output is given by: $Y = n_1f(\phi_1) + n_2g(\phi_2)$, where n_i is the number of students allocated to type i teachers.

The teachers' utility function (common to all teachers) is: $U = w\phi^{-\beta}$, where $0 < \beta < 1$ and w is the wage. Therefore, teachers are always willing to increase the class size by $x\%$ if their wages increase by the same $x\%$.⁸ The aggregate supply type i teachers is: $s_i = A_iU_i$, where A_i is a fixed parameter.

Before we compare the allocation in the “market” system with that in the traditional public system, let us examine the solution of a central planner whose objective is to maximize the total educational output.

3) Central Planner

The task of the central planner would be to fix: ϕ_1 , ϕ_2 , w_1 , and w_2 to maximize the total educational output.

$$\max_{\phi_i, w_i} Y = n_1f(\phi_1) + n_2g(\phi_2)
\tag{2}$$

subject to:

$$R = w_1s_1 + w_2s_2 \text{ and}$$

$$N = s_1\phi_1 + s_2\phi_2$$

This problem can be rewritten as (3), where λ and G are the Lagrange multipliers for the respective constraints:

⁷Aos and Pennucci (2003) review 53 studies that evaluate the impact of the reduction in class size. In all cases, the elasticity of performance with regard to the class size, in absolute terms, is less than 0.15.

⁸DeArmond and Golghaber (2008) show that most teachers in the state of Washington (83%) prefer an increase in wage of \$5,000 per year over a reduction of two students in the classroom, and the two alternatives imply approximately the same increase in costs for the system.

$$\max_{\phi_i, w_i} Z = A_1 w_1 \phi_1^{1-\beta} f(\phi_1) + A_2 w_2 \phi_2^{1-\beta} g(\phi_2) - \lambda (A_1 w_1 \phi_1^{1-\beta} + A_2 w_2 \phi_2^{1-\beta} - N) - G(A_1 w_1^2 \phi_1^{-\beta} + A_2 w_2^2 \phi_2^{-\beta} - R) \quad (3)$$

Assuming the existence of an internal solution to (3), we obtain conditions (4) and (5):

$$\frac{y_1 \varepsilon_{\phi_1} \phi_1}{y_2 \varepsilon_{\phi_2} \phi_2} = \frac{w_1}{w_2} \quad (4)$$

$$\frac{y_1 - \lambda}{y_2 - \lambda} = \frac{y_1 \varepsilon_{\phi_1}}{y_2 \varepsilon_{\phi_2}} \quad (5)$$

Condition (4) states that the Marginal Rate of Substitution between skilled and unskilled teachers is equal to the ratio of their wages, which is the allocation selected by a central planner without constraints.

4) Market Mechanism

Let us now consider the problem of teacher allocation under the “market” arrangement. Let us assume that schools can only charge the amount transferred by the manager for each student as the tuition. Thus, all schools in the education “market” would actually be charter schools. Students maximize learning and may freely choose the teacher they prefer to achieve this goal. Each teacher receives a compensation that is proportional to the number of students in his/her class.

Under the hypothesis that students want to achieve the highest possible grade and have all the necessary information, the equilibrium allocation occurs with $y_1 = y_2$ and $\frac{w_1}{\phi_1} = \frac{w_2}{\phi_2}$, if there is an equilibrium with both types of teachers. Thus, all classes have the same performance and good teachers have more students, therefore earning a proportionately higher wage.⁹

5) Traditional Public System or Constrained Manager

⁹For simplicity, consider that every school consists of only one teacher.

The education manager's task in the traditional public system is to fix ϕ_1 , ϕ_2 , w_1 , and w_2 to maximize its objective function. A feature of public education systems is that the manager faces institutional constraints on employee hiring, compensation, and management decisions. Let us consider an efficient public officer who is subject to two constraints: paying the same wage and fixing the same class size for all teachers.¹⁰ Therefore, if R is the amount of resources to provide education for N homogenous students, with S teachers hired, then $w = \frac{R}{S}$ and $\phi = \frac{N}{S}$.

The hypothesis that the only source of inefficiency arises from the institutional rules imposed on the manager is clearly optimistic. It is reasonable to think that she does not have the knowledge and means to find the conditional optimum. Regardless, the hypothesis of an efficient manager provides an important benchmark. Thus, the manager's decision is restricted to the number of contracted teachers of each type. For simplicity, assume that the objective function of the manager only considers the total educational output.

For the wage w^* , the manager hires all type 1 teachers who are willing to work, but can only hire a portion of type 2 workers. Thus: $S = s_1 + as_2$ ($0 \leq a \leq 1$), $n_1 = \frac{s_1 N}{S}$, $n_2 = \frac{as_2 N}{S}$, $S = s_1(1 + ab)$ and $A_2 = bA_1$, with $b \geq 0$. Without loss of generality, we can set $g(\phi) = \theta f(\phi)$, where θ may depend on ϕ : $0 < \theta(\phi) < 1$.¹¹ Then, the manager's problem is now given as follows:

$$\max_a Y = Nf(\phi) \left[\frac{1+ab\theta}{1+ab} \right] \quad (6)$$

$$\phi = \frac{N}{S}$$

$$S = [(1 + ab)\bar{S}]^{\frac{1}{2-\beta}}$$

$$\bar{S} = A_1 R N^{-\beta}$$

¹⁰ The most common compensation rule for public school teachers is that they are paid on a wage scale, with wages determined entirely by the teachers' educational attainment and experience. Hanushek and Rivikin (2006) argue that this is very different from paying according to the quality of the teacher. It is also unusual to adjust the class size according to the quality of the teacher.

¹¹ When $\theta'(\phi) > 0$, the implication is that good teachers are relatively better at handling small classes. On the other hand, the condition $\theta'(\phi) < 0$ implies that good teachers are relatively better at handling large groups.

By differentiating Y in relation to a and equating the result to zero, we obtain (7), whose solution is given by (8).

$$\frac{dY}{da} = f(\phi) \frac{Nb}{(1+ab)^2} \left[(\theta - 1) - \frac{\delta_\phi}{2-\beta} - ab\theta \frac{(\eta_\phi + \delta_\phi)}{2-\beta} \right] = 0 \quad (7)$$

$$\delta_\phi = \frac{\partial f}{\partial \phi} \frac{\phi}{f} \text{ and } \eta_\phi = \frac{\partial \theta}{\partial \phi} \frac{\phi}{\theta}$$

$$a = \frac{(\theta-1)(2-\beta) - \delta_\phi}{b\theta(\eta_\phi + \delta_\phi)} \quad (8)$$

Note that the elasticity of g in relation to ϕ is given by $\eta_\phi + \delta_\phi$ and therefore $\eta_\phi + \delta_\phi < 0$.¹² To ensure that the condition given by (8) is optimal, it is necessary to ensure that it is met for $0 < a < 1$ and that, at this point, $\frac{d^2Y}{da^2} < 0$. As noted above, the relationship between student performance and class size seems to be very inelastic.¹³ Therefore, if the quality differences between the two types of teachers is large, one would expect that $(1 - \theta)(2 - \beta) + \delta_\phi > 0$, which means that, in (8), we would have $a > 0$. Moreover, when $a = 0$, $\frac{dY}{da} < 0$.

The sign of the slope of $Y(a)$ is given by I:

$$I = \theta(2 - \beta + ab|\varepsilon_{\phi 2}|) - (2 - \beta - |\varepsilon_{\phi 1}|) \quad (9)$$

When $a = 0$, $I < 0$. Assuming that $I(a)$ is a monotonic function in the range: $0 \leq a \leq 1$, we have two situations to consider. The first is that $Y(a)$ decreases throughout the relevant range. In this case, the optimal solution is to hire only type 1 teachers. In the second, there is an inversion of the signal in that range, and therefore, $Y(a)$ has a global minimum in the relevant range. In this case, there are two possible solutions: either the manager in the traditional public system hire only the skilled teachers, or he sets a wage and hire all teachers who are willing to work for them. The solution consists of comparing $Y(0)$ and $Y(1)$ to observe which is higher. The higher the value of b , the greater the chance that $Y(1) > Y(0)$.

¹²In this case, we have $\delta_\phi = \varepsilon_{\phi 1}$ and $(\eta_\phi + \delta_\phi) = \varepsilon_{\phi 2}$.

¹³See footnote 6.

Thus, assuming that $(1 - \theta)(2 - \beta) + \delta_\phi > 0$ and that $I(a)$ is a monotonic function in the relevant range, the system manager chooses $a = 0$ or $a = 1$. Generally, the second solution requires that the stock of type 2 teachers is significantly higher than that of type 1 teachers, provided that b is large enough.

6) Comparisons between Models

6.1. Central Planner versus Market

We have seen that the “market” solution produces performance equality between similar students whereas the central planner maximizes the total educational output (Y). An interesting question is to determine under what conditions the “market” solution is equivalent to the solution of the efficient central planner. From (4) and (5), we observe that this equivalence occurs when at the point of maximum the elasticities are equal ($\varepsilon_{\phi_1} = \varepsilon_{\phi_2}$) and $n_1 + n_2 = N$ is an active constraint, meaning that $\lambda \neq 0$. Under such conditions, the maximization of the total educational output implies that $y_1 = y_2$ and $\frac{w_1}{\phi_1} = \frac{w_2}{\phi_2}$. Such conditions would evidently be satisfied if the production functions have constant and equal elasticities. These conditions would also be met in the case of linear production functions with the same intercept: $y_i = A - b_i\phi_i$, with $b_1 < b_2$. In these cases, the central planner does the same as the “market”. Because teaching in the classroom is subject to significant congestion effects and learning is not interchangeable (a student cannot exchange part of his or her learning with that of another student), only under certain conditions does the “market” solution maximize the total educational output.¹⁴ However, if we think that equality of results is desirable, then it is possible that the “market” solution is superior to the solution that maximizes total educational output.

6.2. Traditional Public System: with and without Charter Schools

¹⁴ If the transfer of a student from classroom A to classroom B improves the student’s performance, then it would occur in the “market” arrangement, even if the total educational output was reduced. The reason is, in that case, the two agents involved in the transaction (the student who transfers and the teacher who accepts him or her) would be in a better situation. If the losses in student performance from classroom B outweigh the gains of the students who remained in A, plus the gain of the student who transfers, such a transfer could be avoided if those who lost with this operation are compensated. Therefore, if learning is interchangeable and there is a method of doing so by the “market”, then the “market” solution would coincide with the maximum total educational output.

We can now compare the public school system run by a constrained manager in two distinct situations: one situation in which the presence of charter schools is permitted and another situation in which it is not permitted. Charter schools receive the same resources per student as the traditional public schools.

The first point to highlight is that charter schools restore the “market” conditions, but do not necessarily eliminate traditional public schools. In this case the public education system is composed of two subsystems: one formed by traditional public schools (traditional public subsystem) and another formed by charter schools (“market” subsystem). With the presence of charter schools, the amount of resources they receive is $\frac{R}{N}n_c$, where n_c is the number of students enrolled in a charter school. We can regard charter schools as teacher cooperatives that, similarly to the traditional public schools, have no costs other than teacher wages. Then, the wage of the traditional public school teacher is given by: $\frac{R(N-n_c)}{NS}$, where S is the number of teachers employed by the traditional public schools.

Charter schools expand whenever there is demand from teachers.¹⁵ Because there is no cost to open a charter school, any teacher can form a new class with students who are willing to take classes with her and receive $\frac{R}{N}$ per student taught from the system administration. Ultimately, each teacher may constitute a charter school.

It should be clear that, in equilibrium, the performance of the students has to be the same across all charter schools. The reason is that students prefer schools where performance is higher, teachers prefer larger classes (because this means higher wages), and there is no constraint on the class size in charter schools. If the student performance is the same in all charter schools, equal-quality teachers receive the same wage and teach to classes of the same size.

Moreover, student performance in charter schools (y_c) cannot be greater than the student performance in the traditional public subsystem, whether students are allocated to a type 1 teacher (y_{1p}) or to a type 2 teacher (y_{2p}). If the performance of a student in the traditional public subsystem was lower than that of a student in a charter school, she would move to the charter. Also, y_{2p} cannot be greater than y_c since, if this occurred,

¹⁵The expansion can occur by both the opening of new charter schools in the system and the expansion of existing charter schools.

students in charter schools would wish to transfer to the traditional public subsystem. In this case, the worst that could happen would be to be allocated to a type 2 teacher. Therefore, if there are type 2 teachers in the two subsystems, then we must have $y_{2p} = y_c$.

However, if $y_{2p} = y_c$, then y_{1p} cannot be greater than y_c . If $y_{2p} = y_c$ and $y_{1p} > y_c$, then students in charter schools would wish to transfer to the traditional public subsystem because they would either improve or remain in the same situation. In turn, if $y_{1p} > y_c$, then type 1 teachers in the traditional public subsystem would gain by moving to a charter school: they would gain more students and more income. Therefore, the presence of charter schools ensure that, in equilibrium, the performance of all students is the same, regardless of the subsystem or the teacher type.

Because class size in the traditional public subsystem does not differ by type of teacher, student performance equality makes it impossible for the traditional public subsystem to employ both types of teachers. Thus, in equilibrium, the traditional public subsystem hires only type 1 or only type 2 teachers. If the traditional public subsystem only hires one type of teacher, then teachers with the same quality have the same class size and the same wage, independent of the subsystem in which they teach. Given the wage rule of the traditional public subsystem, it would also be guaranteed that $\frac{w_1}{\phi_1} = \frac{w_2}{\phi_2}$. Finally, charter schools would eliminate any possibility of rationing teacher jobs. Thus, “market” equilibrium conditions would be re-established.

However, there is not a single equilibrium with regard to the distribution of teachers and students between subsystems. The total specialization, with the traditional public subsystem hiring only type 1 teachers and charter schools hiring only type 2 teachers is a possible equilibrium. The specialization of charter schools in hiring only type 1 teachers and the traditional public subsystem of only hiring type 2 teachers is another possible equilibrium. The situation in which the entire system is operated by charter schools would also be feasible. In short, all situations in which charter schools hire all teachers of a particular type and a ratio (between zero and one) of teachers of the other type would be possible equilibria.

We now consider the situation in which, starting from a system of traditional public schools only, charter schools are allowed. Suppose that individuals change their

position only when the alternative is strictly superior to the current one.¹⁶ The final outcome depends on the initial situation, when only the traditional public system operates. For example, if only good teachers were hired in the initial situation, charter schools would specialize in hiring unskilled teachers and offering smaller class sizes. On the other hand, if both types of teachers were hired in the initial situation, then the final equilibrium would occur with charter schools specializing in hiring skilled teachers and offering larger classes.

Thus, permitting the admission of charter schools restores market equilibrium in the situation where all students have the same purchasing power. In such a situation, there is no difference in student performance. If performance equality is desirable and the total educational output is greater under the “market” organization than under a traditional public system, then permitting the operation of charter schools would undoubtedly improve welfare.

The condition for the total educational output to be greater under the “market” allocation is given by (10):

$$[f(\phi_1^*) - f(\phi^*)] + ab[g(\phi_2^*) - g(\phi^*)] > 0 \quad (10)$$

In (10), ϕ^* is the optimal class size in the situation in which wages and class sizes cannot be differentiated by type of teacher, whereas ϕ_1^* and ϕ_2^* are the class sizes obtained by the “market” rule. Condition (10) depends on the specific forms of both the educational production functions and the teachers’ labor supply functions. However, as discussed in the appendix, condition (10) would be satisfied under fairly mild conditions. Thus, improvements of the school system when charter schools are allowed should be viewed as the rule rather than the exception.

7) Discussion

The central argument of this article is that charter schools operate as a mechanism to circumvent the institutional constraints imposed on the public manager and thus restore the “market” equilibrium. Under certain conditions, the “market” equilibrium is superior

¹⁶This is equivalent to considering the existence of a small fixed cost, which students and teachers would have to bear, to change schools. No one would be willing to bear this cost to transfer from school A to school B if the benefits of school B were the same as those of school A.

to the result obtained by the traditional public system. The paper also highlights the fact that if charter schools could operate freely, then their students would not be expected to learn more than students from traditional public schools. Thus, in this situation it would be incorrect to decide on a policy of charter school expansion based on the difference in average test scores between the two types of schools.

In this scenario, one may ask whether it would be easier simply to remove such restrictions. In this regard, two points should be highlighted. First, it may be easier to pass legislation on charter schools than to change rules on hiring and resource allocation of traditional public schools. Second, the hypothesis that the public manager does not reach the optimal solution only because of the institutional constraints is very optimistic. It would be more reasonable to assume that, even without such constraints, the manager would not have the knowledge and means to find the optimal solution.

A key point in our argument is that the constraints imposed on the manager of the public education system may lead her to restrict the hiring of unskilled teachers. Under such circumstances, permitting charter schools could lead to hiring these teachers at a lower wage and allow them to teach in smaller classes. On the other hand, the public system may have teachers of different skill levels. In this case, the skilled teachers would be interested in teaching in larger classes and earning proportionately more, which would be possible by the introduction of charter schools.

For simplicity, we consider only two types of teachers, finding that charter schools specialize in hiring unskilled teachers who work with smaller class sizes or in hiring the skilled teachers who work with larger class sizes. However, considering three groups of teachers may provide a better description of reality. Jacob and Lefgren (2008) show that principals can identify teachers who fall into the extremes of the quality distribution (measured as effectiveness in increasing the scores of students on a standardized test), but have difficulty in discriminating teachers in the middle of the distribution. With three types of teachers, the equilibrium may require charter schools to specialize in operating with both the most skilled and the least skilled teachers.

While the rise of teacher efficiency in charter schools has been highlighted before by charter school proponents, this article focuses on the process of hiring and allocation of teachers and its effect on overall efficiency. We assume that teachers' performance does not depend on the type of school in which they teach. Naturally, competition for students may provide an incentive for teachers to put forth more effort and if there is a

lack of incentives in traditional public schools, then this would be an additional reason to defend the introduction of charter schools.

The entire analysis assumes that there is complete freedom for charter schools to open and expand and that they are endowed with the same amount of resources per student as the traditional public schools, which does not necessarily correspond with reality. According to the Center for Education Reform (2014), charter schools have fewer resources per student (see also Batdorff et al., 2014). In addition, states often impose restrictions on the opening and expansion of charter schools (Terry and Kim, 2009). These facts limit further expansion of charter schools in the United States.

Moreover, some important Charter Management Organizations have excellent performance and excess demand, but do not expand the number of vacancies, as this paper predicts. However, many of them attract significant resources from private donations (see Furgeson et al., 2012) and if good performance is important for the receipt of private resources, then this would be a reason for not expanding. Regardless, these schools are not representative of all charter schools. According to Furgeson et al. (2012), the charter schools belonging to Charter Management Organizations represent approximately 16% of all charter schools in the United States.

If the “market” solution is indeed more efficient, then an alternative to the public manager would be to impose rules that replicate the market mechanism (the quasi-markets), but that discussion is beyond the scope of this article.

Finally, this paper considers that students are homogeneous or, at least, that the distribution of skills is not different between schools and classes. An important extension would be to consider the case in which different teachers (or schools) systematically receive students with different levels of learning potential, which can occur, for example, if the students’ profiles vary according to the region of residence and students are required to attend a school in the area where they live.

References

Abdulkadiroglu, A., Angrist, J., Cohodes, S., Dynarski, S., Fullerton, J. Kane, T. and Pathak, P. (2009). *Informing the Debate: Comparing Boston’s Charter, Pilot and Traditional Schools*. Boston: Boston Foundation.

Angrist, J. D., Dynarski, S. M., Kane, T. J., Pathak, P. A. and Walters, C. R. (2012). Who Benefits from KIPP. *Journal of Policy Analysis and Management*, vol.31: 837-860.

Angrist, J. D. and Lavy, V. (1999). Using Maimonides' Rule to Estimate the Effect of Class Size on Scholastic Achievement. *Quarterly Journal of Economics*, vol. 114: 533-575.

Angrist, J. D., Pathak, P. A. and Walters, C. R. (2013). Explaining Charter School Effectiveness. *American Economic Journal: Applied Economics*, vol.5: 1-27.

Aos, S. and Pennucci, A. (2013). *K-12 Class Size Reductions and Student Outcomes: A Review of the Evidence and Benefit-Cost Analysis*. Document No. 13-01-2201. Olympia, WA: Washington State Institute for Public Policy.

Batdorff, M., Maloney, L., May, J. F., Speakman, S. T., Wolf, P. J. and Cheng, A. (2014). *Charter School Funding: Inequity Expands*. School Choice Demonstration Project, Department of Education Reform, University of Arkansas.

Bifulco, R. and Ladd, H. F. (2006). The Impact of Charter Schools on Student Achievement: Evidence from North Carolina. *Education Finance and Policy*, vol. 1: 50-90.

Booker, T. K., Gilpatric, S., Gronberg, T. and Jansen, D. (2007). The Impact of Charter School Attendance on Student Performance. *Journal of Public Economics*, vol. 91: 849-876.

Center for Education Reform (2014). *Annual Survey of America's Charter Schools 2014*. Washington, DC: Center for Education Reform.

Chudowsky, N. and Ginsburg, A. (2012). *Who attends charter schools and how are those students doing?* Washington, DC: National Assessment Governing Board.

Clark, M. A., Gleason, P., Tuttle, C. C. and Silverberg, M. K. (2011). *Do Charter Schools Improve Student Achievement? Evidence from a National Randomized Study*. Princeton, NJ: Mathematica Policy Research.

DeArmond, M. and Golghaber, D. (2008). *A Leap of Faith: Redesigning Teacher Compensation*, Center on Reinventing Public Education. University of Washington.

Dobbie, W. and Fryer Jr., R. G. (2011). Are High-Quality Schools Enough to Increase Achievement Among the Poor? Evidence from the Harlem Children's Zone. *American Economic Journal: Applied Economics*, vol. 3: 158-187

Epple, D., Romano, R. E. and Urquiola, M. (2015). *School Vouchers: A Survey of the Economics Literature*. NBER Working Paper Series No. 21523. Cambridge, MA: National Bureau of Economic Research.

Epple, D., Romano, R. E. and Zimmer, R. (2015). *Charter Schools: A Survey of Research on their Characteristics and Effectiveness*. NBER Working Paper Series No. 21523. Cambridge, MA: National Bureau of Economic Research.

- Friedman, M. (1962). The Role of Government in Education. In *Capitalism and Freedom*, Chapter 6. Cambridge University Press, Cambridge, MA.
- Furgeson, J., Gill, B., Haimson, J., Killewald, A., McCullough, M., Nichols-Barrer, I., Teh, B., Savitz, N. V., Bowen, M., Demeritt, A., Hill, P. and Lake, R. (2012). *Charter-School Management Organizations: Diverse Strategies and Diverse Student Impacts*. Cambridge, MA: Mathematica Policy Research.
- Goldring, R., Gray, L. and Bitterman, A. (2013). *Characteristics of Public and Private Elementary and Secondary School Teachers in the United States: Results from the 2011–12 Schools and Staffing Survey*. Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education.
- Hanushek, E. A., Kain, J. F., Rivkin, S. G. and Branch. G. F. (2007). Charter School Quality and Parental Decision Making with School Choice. *Journal of Public Economics*, vol. 91: 823-848.
- Hanushek, E. A. and Rivkin, S. G. (2006). Teacher Quality. In Hanushek, E. A and Welch, F. (eds.) *Handbook of the Economics of Education*, pp 1051-1078. Amsterdam: North Holland.
- Hoxby, C. M., Murarka, S. and Kang, J. (2009). *How New York City's Charter Schools Affect Student Achievement: August 2009 Report*. Second Report in Series. Cambridge, MA: New York City Charter Schools Evaluation Project.
- Hoxby, C. M., and Rockoff, J. E. (2005). Findings from the City of Big Shoulders. *Education Next*, vol. 5: 52-58.
- Jacob, B. A. and Lefgren, L. (2008). Can Principals Identify Effective Teachers? Evidence on Subjective Performance Evaluation in Education. *Journal of Labor Economics*, vol. 26: 101-136.
- Krueger, A. B. (1999). Experimental Estimates of Education Production Functions. *Quarterly Journal of Economics*, vol. 114: 497-532.
- Sass, T. R. (2006). Charter Schools and Student Achievement in Florida. *Education Finance and Policy*, vol. 1: 91-122.
- Schanzenbach, D. W. (2010). The Economics of Class Size. In Brewer, D. J. and McEwan, P. J. (eds.) *Economics of Education*, pp 183-190. San Diego, CA: Elsevier.
- Terry, B. D. and Kim, J. (2009). *Charter School Caps*. Texas Public Policy Foundation.
- Zimmer, R., Gill, B., Booker, K., Lavertu, S., Sass, T. R. and Witte, J. (2009). *Charter Schools in Eight States: Effects on Achievement, Attainment, Integration, and Competition*. Santa Monica, CA: RAND Corporation.

Appendix - Market Efficiency with respect to the Constrained Manager

As observed in the main text, total educational output will be greater under the “market” allocation as compared to traditional public system if:

$$[f(\phi_1^*) - f(\phi^*)] + ab[g(\phi_2^*) - g(\phi^*)] > 0 \quad (10)$$

Under reasonable assumptions, one would expect that the solution for the traditional public system involves $a = 0$ or $a = 1$. Thus, let us analyze these cases. First, consider that $a = 0$, which means that the condition (10) is reduced to $[f(\phi_1^*) - f(\phi^*)] > 0$. This condition must be met unless no type 2 teacher is hired when the “market” system is in operation.

Starting from a traditional public system whose optimal solution is to hire only type 1 teachers and allowing the participation of charter schools, no type 2 teacher would be hired if $f(\phi^* - 1) > g(1)$. In this situation, the traditional public system would not differ from the “market” system, and we would have $[f(\phi_1^*) - f(\phi^*)] = 0$.

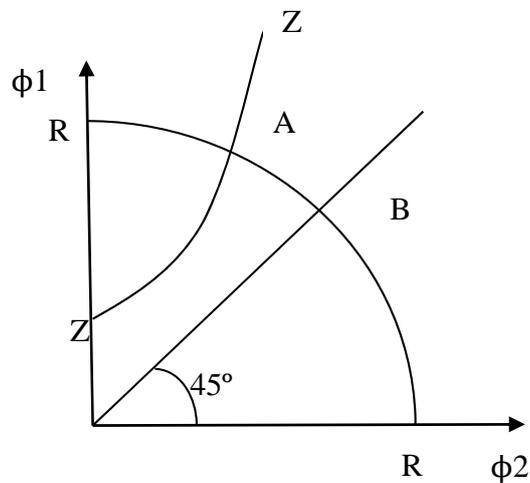
On the other hand, assuming that $f(\phi^* - 1) < g(1)$, there would be type 2 teachers willing to work earning $\frac{R}{N}$ per student and providing a better education than the current teachers. Thus, if the “market” system is implemented, there would be students willing to abandon type 1 teachers and transfer to type 2 teachers. If the number of type 1 teachers remains constant, compliance with the above condition would be straightforward because the reduction in class size improves performance. However, a smaller class size, with the consequent wage reduction, reduces the supply of type 1 teachers. Ultimately, however, the class size should be smaller because, otherwise, the supply of type 1 teachers could not be reduced. Thus, we have $\phi_1^* < \phi^*$ and $f(\phi_1^*) > f(\phi^*)$.

Let us assume now that $a = 1$. In this situation, the wage paid to teachers, both in the “market” solution and under the rule of the traditional public administration, is given by: $\frac{R}{N} \phi_i$ ($i = 1$ or 2). Then, given that $s_1 \phi_1 + s_2 \phi_2 = N$, we obtain (11).

$$\phi_1 = \left[\frac{N^2}{A_1 R} - b \phi_2^{2-\beta} \right]^{\frac{1}{2-\beta}} \quad (11)$$

Equation (11) gives all possible combinations of ϕ_1 and ϕ_2 that deplete all of the available resources and comply with the wage rule described above. It has slope $-b \left(\frac{\phi_2}{\phi_1}\right)^{1-\beta}$ and is represented by the curve R in Figure 1.

Figure1 – Possibilities related to Classes Sizes



In addition to meeting condition (11), the “market” rule requires that $f(\phi_1) = g(\phi_2)$. The combinations of ϕ_1 and ϕ_2 that meet this restriction are given by (12) and are represented by the Z curve in Figure 1. The slope of (12) is given by $\frac{g'}{f'}$ and therefore is dependent on the functional forms of f and g . The market solution is represented by the point A.

$$\phi_1 = f^{-1}[g(\phi_2)] \tag{12}$$

In Figure 1, point B represents the situation in which $\phi_1 = \phi_2$ and condition (11) is met. Thus, it represents the solution of the traditional public system. Therefore, what we need to know is whether the total educational output is greater at point A or at point B.

To analyze this, it would be interesting to find the condition that maximizes the educational output, given that (11) is met. This condition is given by (13).

$$\max_{\phi_1, \phi_2} Y = f(\phi_1)A_1 \frac{R}{N} \phi_1^{2-\beta} + g(\phi_2)A_2 \frac{R}{N} \phi_2^{2-\beta} \quad (13)$$

subject to:

$$N^2 = A_1 R \phi_1^{2-\beta} + A_2 R \phi_2^{2-\beta}$$

Assuming the existence of an internal solution in (13), the first-order condition is given by (14).

$$y_1(2 - \beta + \varepsilon_{\phi_1}) = y_2(2 - \beta + \varepsilon_{\phi_2}) \quad (14)$$

Defining $\delta_\phi = \frac{\partial f}{\partial \phi} \frac{\phi}{f}$ and $\eta_\phi = \frac{\partial \theta}{\partial \phi} \frac{\phi}{\theta}$ in (14) we have $\varepsilon_{\phi_1} = \delta_{\phi_1}$ and $\varepsilon_{\phi_2} = \eta_{\phi_2} + \delta_{\phi_2}$. Note, for example, that, if the elasticities are constant and equal ($\varepsilon_{\phi_1} = \varepsilon_{\phi_2} = \varepsilon$), then the “market” solution is optimal, as would be expected.¹⁷

If we assume that $\delta'_\phi \geq 0$ and $\theta'(\phi) \leq 0$, we would have $|\varepsilon_{\phi_1}| \leq |\varepsilon_{\phi_2}|$, which means that the optimal solution would be at point A (when $\delta'_\phi = 0$ and $\theta'(\phi) = 0$) or at a point to its left. Thus, condition (10) would be guaranteed. The term $\delta'_\phi > 0$ means that, in absolute terms, the elasticity of performance in relation to class size is reduced when the class size increases, while the term $\theta'(\phi) < 0$ means that good teachers are relatively better at handling large groups, which means that, for very small classes, the type of teacher would not make much difference.¹⁸

Even if these assumptions do not hold, condition (10) may still be met. The smaller the value of β and the more inelastic the student performance in relation to the group size (ε_{ϕ_i}), the greater the chance that (10) will hold.¹⁹ Under such circumstances, $\frac{(2-\beta+\varepsilon_{\phi_1})}{(2-\beta+\varepsilon_{\phi_2})}$ would not be expected to deviate too far from the unit, which means that the optimal solution would be near point A. Thus, it would be expected that the total educational

¹⁷By assumption: $(2 - \beta + \varepsilon_{\phi_i}) > 0$. By defining $z_i = y_i(2 - \beta + \varepsilon_{\phi_i})$, a condition sufficient for (14) to be a maximum condition is that $\frac{\partial z_i}{\partial \phi_i} < 0$.

¹⁸The condition that $\theta'(\phi) < 0$ implies $\eta_\phi < 0$.

¹⁹See footnote 6.

output would be greater under the “market” arrangement than under the traditional public arrangement.