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Abstract

We develop a multisector model in which capital and labor are free to move across firms within each sector, but cannot move across sectors. To isolate the role of sectoral specificity, we compare our model with otherwise identical multisector economies with either economy-wide or firm-specific factor markets. Sectoral factor specificity generates within-sector strategic substitutability and tends to induce across-sector strategic complementarity in price setting. Our model can produce either more or less monetary non-neutrality than those other two models, depending on parameterization and the distribution of price rigidity across sectors. Under the empirical distribution for the U.S., our model behaves similarly to an economy with firm-specific factors in the short-run, and later on approaches the dynamics of the model with economy-wide factor markets. This is consistent with the idea that factor price equalization might take place gradually over time, so that firm-specificity may serve as a reasonable short-run approximation, whereas economy-wide markets are likely a better description of how factors of production are allocated in the longer run.

JEL classification codes: E22, J6, E12

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1 Introduction

Much of the monetary economics literature tries to make sense of the extent of monetary non-neutrality that is apparent in the data. An important part of this literature does so by resorting to models in which prices (and sometimes wages) are sticky. A problem with bare bone versions of these models is that the degree of price rigidity required to generate substantial non-neutrality is at odds with the microeconomic evidence on the frequency of price changes. However, since Ball and Romer (1990) and Kimball (1995), it is well-known that large real rigidities – which can induce strategic complementarities in price-setting decisions – can generate substantial endogenous persistence in the real effects of monetary shocks, and thus help bridge this gap.[1]

In a series of contributions to our understanding of the sources of real rigidities, Woodford (2003, 2004, 2005) argues forcefully that factor specificity matters. In particular, Woodford (2005) develops a model in which both capital and labor are specific to firms – i.e., they cannot move freely from one firm to another.[2] He shows that factor specificity at the firm level is a powerful source of real rigidities. Other papers in the literature, such as Sveen and Weinke (2007a) and Altig et al. (2011), also find that firm-specific capital is an important ingredient for understanding the monetary transmission mechanism.

The assumption of firm-level specificity contrasts sharply with the (usually unstated) assumption that factors of production can move freely across firms, as in the Real Business Cycle literature. Under standard assumptions about preferences and technology, such economy-wide factor markets tend to induce strategic substitutability in price setting (e.g., Woodford 2003, chap. 3), and thus generate a small degree of monetary non-neutrality (Chari et al. 2000).

The two aforementioned assumptions about factor markets are, to some extent, unrealistic. It is likely that factor price equalization takes place gradually over time, so that firm-specificity might be a reasonable short-run approximation, whereas economy-wide markets might be a better description of how factors of production are allocated in the longer run.

In this paper, we study whether the *nature* of factor specificity matters. To that end, we develop a multisector model in which capital and labor are free to move across firms *within* each sector, but cannot move *across* sectors – i.e., factors of production are sector-specific. To isolate the role of sectoral specificity, we compare our model with otherwise identical multisector economies with either economy-wide or firm-specific factor markets.

It turns out that it matters a great deal whether factor markets are specific at the firm or at the sector level. Sectoral factor specificity generates *within-sector strategic substitutability* in pricing

[1] Other such mechanisms have to do with, for example, information frictions and heterogeneity in price rigidity.

[2] To be precise, Woodford's (2003, 2005) models feature industry-specific labor coupled with assumptions that make it mathematically equivalent to a particular model with firm-specific labor markets, as will become clear subsequently.

decisions. This tends to reduce the degree of monetary non-neutrality relative to the case of firm-specific factors. At the same time, sectoral relative price movements generate distributional effects that induce strategic complementarity (or weaken strategic substitutability) in pricing decisions *across sectors* – relative to the model with economy-wide factor markets. As a result of these forces, our sector-specific factor model can produce either more or less monetary non-neutrality than those other two models, depending on parameterization and the distribution of price rigidity across sectors. A calibrated version of our model that matches the empirical distribution of price stickiness for the U.S. behaves similarly to an economy with firm-specific factors in the short-run, and later on approaches the dynamics of the model with economy-wide factor markets.

As shown by Woodford (2005), firm-specific factors can be a powerful source of real rigidities, and hence strengthen strategic complementarities in firms' pricing decisions. To understand the mechanism at work, consider first the case of economy-wide factor markets. Furthermore, for ease of exposition, let us focus on a parameterization for preferences and technology such that firms' marginal costs can be thought of as being exogenous. Hence, with constant desired mark-ups, firms' pricing decisions do not interact – a case that we refer to as featuring strategic neutrality in price setting. In that case, a given nominal disturbance will have some effect on real variables due to nominal price rigidities only.

Switching to an economy with firm-specific factor markets, but keeping the same parameterization underlying the environment described in the previous paragraph, leads to strategic complementarities in pricing decisions – both within and across sectors. The mechanism at work is the following. Consider an increase in nominal aggregate demand, in response to which all firms will eventually raise their prices. A firm that increases its price at some point in time faces relatively less demand for its variety, when compared to other firms in the same sector. Moreover, as long as there is some substitutability between varieties of consumption goods produced in different sectors, demand will also shift away toward firms in other sectors in response to that price increase. This relative drop in demand will lead adjusting firms to deploy relatively fewer production inputs, putting downward pressure on firm-specific factor prices – relative to the case in which these prices are determined at the economy-wide level, and are hence insensitive to individual firms' demands. As a result, under firm-specific factor markets, adjusting firms will tend to increase their prices by less in response to an increase in nominal aggregate demand.

Underlying the analysis of the previous paragraph is the fact that, under the assumed parameterization for preferences and technology, the model with firm-specific factor markets induces complementarities in price setting decisions both within and across sectors. This can be seen clearly if we resort to the usual comparative statics analysis that focuses on how the desired price of an atomistic flexible-price firm responds to exogenous changes in the aggregate price level or to exogenous changes in the price level in its sector.

In response to, say, an exogenous increase in the overall level of prices, nominal marginal costs for the firm at hand increase. The reason is that the assumed aggregate price increase makes the firm's sector varieties relatively cheaper. This increases demand for its variety, and hence its demand for production inputs. The other forces that would affect the firm's nominal marginal cost cancel out exactly, due to the aforementioned parameterization that leads to strategic neutrality in price setting in the model with economy-wide factor markets.[3]

Likewise, an exogenous increase in the overall level of prices in the sector in which the firm at hand operates also increases its nominal marginal cost. As in the previous paragraph, the reason is that the variety that the firm produces becomes relatively cheaper.[4]

The bottom line is that the model with firm-specific factor markets features strategic complementarities in price-setting decisions both within and across sectors. As is well known, in the presence of nominal rigidities, these complementarities tend to produce larger real effects of nominal disturbances when compared to a similar model with strategic neutrality in price setting.

Consider now the model with sector-specific factor markets. As in the previous model, an exogenous increase in the overall level of prices in the economy increases nominal marginal costs for the atomistic flexible-price firm under consideration. This is the case because the increase in the aggregate price level makes varieties in that firm's sector relatively cheaper. This shifts demand in favor of that sector, increasing demand for production inputs and putting upward pressure on factor prices – which, by assumption, are determined at the sectoral level. Hence, the model with sector-specific factor markets also features across-sector complementarities in price-setting decisions.

In sharp contrast, an exogenous increase in the overall level of prices in the sector in which the firm at hand operates decreases nominal marginal costs for that firm. The reason is that the increase in the relative price of the sector (when compared with prices elsewhere in the economy) shifts demand away from it, decreasing the sector's demand for production inputs and putting downward pressure on factor prices. That is, this model features *within-sector substitutability* in price-setting decisions. As a result of different patterns of pricing interactions within and across sectors, our model with sector-specific factor markets can produce rich aggregate dynamics, as we discuss in Sections 3 and 4.

The differences in aggregate dynamics implied by the different assumptions on factor specificity can also be understood through the lens of the underlying New Keynesian Phillips curves. We

[3] See Section 3 for details.

[4] There is also an effect working in the opposite direction. The increase in the overall level of prices in a sector relative to prices elsewhere in the economy lowers overall demand for varieties in that sector. However, this effect is dominated by the effect of substitution within the sector in which the firm under consideration operates – as long as the elasticity of substitution between varieties within any given sector exceeds the elasticity of substitution between varieties across sectors (which is a plausible assumption).

explicitly derive Phillips curves in our multisector economies with endogenous capital accumulation, under the three alternative assumptions regarding factor specificity.[5]

While the assumption that factors cannot move across sectors is also extreme, our model is motivated by existing empirical evidence that both capital and labor have an important degree of sector (or industry) specificity. For example, Ramey and Shapiro (1998) find that the flow of capital across firms within the same industry is indeed large, while Ramey and Shapiro (2001) provide evidence of significant sectoral specificity of capital, based on an industry case study. The empirical evidence that labor reallocation across sectors/industries is more limited than within sectors is shown in Davis and Haltiwanger (1992) and Parent (2000). More recently, Hobijn (2012) finds similar results. Autor et al. (2014) show that the intensity of labor reallocation within- versus across-sectors correlates with the level of wages. In particular, looking at the effects of increasing import competition, they find that low-wage workers reallocate primarily within industry, while high-wage workers appear to switch sectors more easily.

Different forms of input specificity can also be justified on theoretical grounds. In a series of contributions, Caballero (2007), and Caballero and Hammour (1996, 1998, 2000) discuss the possible sources and macroeconomic implications of input specificity. They show that specificity contributes to the slow adjustment of macroeconomic aggregates to shocks in the short and medium run, with resources being underutilized, production suffering from "technological sclerosis," and recessions being excessively sharp in the transition to a new steady state.

Section 2 presents the reference model of our multisector economy with sector-specific factors of production. It also presents the otherwise identical multisector models with either economy-wide or firm-specific factor markets. Section 3 analyses in detail the different patterns of pricing interactions produced by the three models, and presents the underlying new Keynesian Phillips curves. Section 4 follows with a quantitative analysis of the effects of monetary shocks under the three types of factor specificity. The last section concludes.

2 Three models of factor specificity

In this section, we consider three alternative sticky-price dynamic stochastic general equilibrium (DSGE) models that differ in their assumptions about factor mobility. In all models, identical infinitely-lived consumers supply labor and capital to intermediate firms that they own, invest in a complete set of state-contingent financial claims, and consume a final good. The latter is produced by competitive firms that bundle varieties of intermediate goods. The monopolistically competitive intermediate firms that produce these varieties are divided into sectors that differ in their frequency

[5] For an analysis of the implications of labor specificity for the NKPC in one-sector models without capital accumulation, see the recent survey by Leahy (2011).

of price changes. Labor and capital are the only variable inputs in the production of intermediate goods. In the first model, presented in Section 2.1, we assume that these inputs can be reallocated freely across firms in the same sector but cannot flow across sectors, i.e., factors are sector-specific. Sections 2.2 and 2.3 consider two alternative cases in which factors of production are, respectively, firm-specific or can move freely across firms and sectors ("economy-wide" factor markets).

2.1 Sector-specific factors[6]

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{s=1}^{S} \omega_s \frac{N_{s,t}^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + P_t I_t + E_t \left[\Theta_{t,t+1} B_{t+1} \right] \leq \sum_{s=1}^{S} W_{s,t} N_{s,t} + B_t + T_t + \sum_{s=1}^{S} Z_{s,t} K_{s,t},$$

the law of motion for the stocks of sector-specific capital:

$$\begin{aligned} K_{s,t+1} &= (1-\delta) K_{s,t} + \Phi \left(I_{s,t}, K_{s,t} \right) I_{s,t}, \forall\ s \\ I_{s,t} &\geq 0, \forall s \end{aligned}$$

and a standard "no-Ponzi" condition. E_t denotes the time-t expectations operator, C_t is consumption of the final good, $N_{s,t}$ denotes total labor supplied to firms in sector s, $W_{s,t}$ is the associated nominal wage rate, and ω_s is the relative disutility of supplying labor to sector s.[7] $I_{s,t}$ denotes investment in sector-s capital, $I_t \equiv \sum_{s=1}^{S} I_{s,t}$, $K_{s,t}$ is capital supplied to firms in sector s, and $Z_{s,t}$ is the associated nominal return on capital. The final good can be used for either investment or consumption, and sells at the nominal price P_t. B_{t+1} accounts for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of $t+1$. Under complete financial markets, agents can choose the value of B_{t+1} for each possible state of the world at all times, subject to the no-Ponzi condition and the budget constraint. T_t stands for profits received from intermediate firms. The absence of arbitrage implies the existence of a nominal stochastic discount factor $\Theta_{t,t+1}$ that prices in period t any financial asset portfolio with state-contingent payoff B_{t+1} at the beginning

[6] Woodford sometimes refers to his assumption regarding labor markets as involving industry-specific (e.g., Woodford 2003, chapter 3) or sector-specific (e.g., Woodford 2005) labor markets. This is not the same as our assumption of sector-specificity. In Woodford's work, an industry (or sector) is characterized by fully synchronized price setting, so that his assumption is mathematically equivalent to firm-specific labor markets – as long as firms behave competitively and do not try to exploit their monopsony power. In contrast, in our model, price changes are asynchronized within each sector.

[7] As in Carvalho and Lee (2011), relative labor disutility parameter is only used to obtain a symmetric steady state, and simplify the algebra. It does not play a role in any of our findings. Derivation details are available upon request.

of period $t+1$.[8] Finally, β is the time-discount factor, σ^{-1} denotes the intertemporal elasticity of substitution, γ^{-1} is the Frisch elasticity of labor supply, δ is the rate of depreciation, and $\Phi(.)$ is the adjustment-cost function. We follow Chari et al. (2000), and assume $\Phi(I_{s,t}, K_{s,t})$ takes the following form:

$$\Phi(I_{s,t}, K_{s,t}) = \Phi\left(\frac{I_{s,t}}{K_{s,t}}\right) = 1 - \frac{1}{2}\kappa \frac{\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2}{\frac{I_{s,t}}{K_{s,t}}},$$

which is convex and satisfies $\Phi(\delta) = 1$, $\Phi'(\delta) = 0$, and $\Phi''(\delta) = -\frac{\kappa}{\delta}$.

The first-order conditions for consumption and labor are:

$$\frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} = \frac{\beta^l}{\Theta_{t,l}} \frac{P_t}{P_{t+l}},$$

$$\frac{W_{s,t}}{P_t} = \omega_s N_{s,t}^{\gamma} C_t^{\sigma}, \ \forall \ s.$$

Consumers' allocation of sectoral investment $I_{s,t}$ and capital $K_{s,t+1}$ yields:

$$Q_{s,t} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\frac{Z_{s,t+1}}{P_{t+1}} + Q_{s,t+1} \left[(1-\delta) + \Phi'\left(\frac{I_{s,t+1}}{K_{s,t+1}}\right) \left(\frac{I_{s,t+1}}{K_{s,t+1}}\right)^2 \right] \right) \right\},$$

$$Q_{s,t}\left(\Phi'\left(\frac{I_{s,t}}{K_{s,t}}\right) \frac{I_{s,t}}{K_{s,t}} + \Phi\left(\frac{I_{s,t}}{K_{s,t}}\right) \right) = 1,$$

where $Q_{s,t}$ denotes Tobin's q for sector s.

The solution must also satisfy a transversality condition:

$$\lim_{l \to \infty} E_t\left[\Theta_{t,l} B_l\right] = 0.$$

A representative competitive firm produces the final good, which is a composite of varieties of intermediate goods. Monopolistically competitive firms produce such varieties. The latter firms are divided into sectors indexed by $s \in \{1, ..., S\}$, each featuring a continuum of firms. Sectors differ in the degree of price rigidity, as we detail below. Overall, firms are indexed by their sector s, and are further indexed by $j \in [0,1]$. The distribution of firms across sectors is given by sectoral weights $f_s > 0$, with $\sum_{s=1}^{S} f_s = 1$.

The final good is used for both consumption and investment and is produced by combining the intermediate varieties according to the technology:

$$Y_t = \left(\sum_{s=1}^{S} f_s^{\frac{1}{\eta}} Y_{s,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

[8] To avoid cluttering the notation, we omit explicit reference to the different states of nature.

$$Y_{s,t} = \left(f_s^{\frac{\theta-1}{\theta}} \int_0^1 Y_{s,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where Y_t is the final good, $Y_{s,t}$ is the aggregation of sector-s intermediate goods, and $Y_{s,j,t}$ is the variety produced by firm j in sector s. The parameters $\eta \geq 0$, and $\theta > 1$ are, respectively, the elasticity of substitution across sectors, and the elasticity of substitution within sectors.

The representative final-good-producing firm solves:

$$\begin{aligned} &\max \ P_t Y_t - \sum\nolimits_{s=1}^{S} f_s \int_0^1 P_{s,j,t} Y_{s,j,t} dj \\ &s.t. \ \ (1)\text{-}(2), \end{aligned}$$

which yields as first-order conditions, for $j \in [0,1]$ and $s = 1, ..., S$:

$$Y_{s,j,t} = f_s \left(\frac{P_{s,j,t}}{P_{s,t}} \right)^{-\theta} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t.$$

The price indices are given by:

$$\begin{aligned} P_t &= \left(\sum\nolimits_{s=1}^{S} f_s P_{s,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \\ P_{s,t} &= \left(\int_0^1 P_{s,j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \end{aligned}$$

where P_t is the price of the final good, $P_{s,t}$ is the price index of sector-s intermediate goods, and $P_{s,j,t}$ is the price charged by firm j from sector s.

Monopolistically competitive firms produce varieties of the intermediate good by employing capital and labor. For analytical tractability, we assume that intermediate firms set prices as in Calvo (1983). The frequency of price changes varies across sectors, and it is the only source of (ex-ante) heterogeneity. While in reality sectors certainly differ in many dimensions other than heterogeneity in price stickiness, this assumption suits our purposes for the following reasons. First, nominal rigidity is of interest, because our goal is to contribute to the literature on factor specificity as a source of real rigidities that can amplify/propagate monetary shocks (e.g., Chari et al. 2000 and Woodford 2005). In this context, allowing for heterogeneity in price stickiness is natural, because of the ample literature documenting that this is empirically important, and that it matters for aggregate dynamics.[9] In addition, in our model, any two different sectors that have the same degree of price rigidity will respond to aggregate shocks in the exact same way.[10] Hence, our partition of the economy in terms of price rigidity may encompass arbitrarily many "subsectors" within each sector.

In each period, each firm j in sector s changes its price independently with probability α_s. At

[9] See, for example, the survey of Klenow and Malin (2010).

[10] This would possibly cease to be the case if sectors differed in other dimensions as well.

each time a firm j from sector s adjusts its price, it chooses $X_{s,j,t}$ to solve:

$$\begin{aligned} \max\ E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \left[\begin{array}{c} X_{s,j,t} Y_{s,j,t+l} + \\ -W_{s,t+l} N_{s,j,t+l} - Z_{s,t+l} K_{s,j,t+l} \end{array} \right] \\ s.t. \qquad Y_{s,j,t} = f_s \left(\frac{P_{s,j,t}}{P_{s,t}}\right)^{-\theta} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t \\ Y_{s,j,t} = F\left(K_{s,j,t} N_{s,j,t}\right) = \left(K_{s,j,t}\right)^{1-\chi} \left(N_{s,j,t}\right)^{\chi}, \end{aligned} \tag{3}$$

where χ is the elasticity of output with respect to labor.

Optimal price setting implies:

$$X_{s,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{s,t+l} \left(\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1}\right)^{-1} W_{s,t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{s,t+l}},$$

where:

$$\Lambda_{s,t} = f_s \left(\frac{1}{P_{s,t}}\right)^{-\theta} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t. \tag{4}$$

From cost-minimization, real marginal cost can be expressed as:

$$MC_{s,j,t} = MC_{s,t} = \frac{1}{\chi^{\chi}\left(1-\chi\right)^{1-\chi}} \left(\frac{W_{s,t}}{P_t}\right)^{\chi} \left(\frac{Z_{s,t}}{P_t}\right)^{(1-\chi)}. \tag{5}$$

Note that marginal costs are equalized only within sectors. This is a direct implication of the assumption of sectoral capital and labor markets. Finally, under that assumption, the market-clearing conditions for capital and labor are:

$$\begin{aligned} K_{s,t} &= f_s \int_0^1 K_{s,j,t} dj,\ \forall\ s, \\ N_{s,t} &= f_s \int_0^1 N_{s,j,t} dj,\ \forall\ s. \end{aligned}$$

2.2 Firm-specific factors

We now consider a variant of the previous model in which production inputs are specific at the firm level. This version of the model generalizes Woodford (2005) to a multisector economy.[11] It requires that we reformulate the consumers' and intermediate firms' problems. The maximization problem of final goods firms remains the same as in the model with sectoral factor markets.

[11] Other papers that model capital as firm-specific are Altig et al. (2011), Sveen and Weinke (2007a), and Sveen and Weinke (2007b).

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{s=1}^{S} f_s \int_0^1 \frac{N_{s,j,t}^{1+\gamma}}{1+\gamma} dj \right),$$

subject to the flow budget constraint:

$$P_t C_t + P_t I_t + E_t \left[\Theta_{t,t+1} B_{t+1} \right] \leq \sum_{s=1}^{S} f_s \int_0^1 W_{s,j,t} N_{s,j,t} dj + B_t + T_t + \sum_{s=1}^{S} f_s \int_0^1 Z_{s,j,t} K_{s,j,t} dj,$$

the law of motion for the stocks of firm-specific capital:

$$\begin{aligned} K_{s,j,t+1} &= (1-\delta) K_{s,j,t} + \Phi \left(I_{s,j,t}, K_{s,j,t} \right) I_{s,j,t}, \forall\ s, j \\ I_{s,j,t} &\geq 0, \forall\ s, j, \end{aligned}$$

where $\Phi \left(I_{s,j,t}, K_{s,j,t} \right)$ now takes the form:

$$\Phi \left(I_{s,j,t}, K_{s,j,t} \right) = \Phi \left(\frac{I_{s,j,t}}{K_{s,j,t}} \right) = 1 - \frac{1}{2} \kappa \frac{\left(\frac{I_{s,j,t}}{K_{s,j,t}} - \delta \right)^2}{\frac{I_{s,j,t}}{K_{s,j,t}}},$$

and a standard "no-Ponzi" condition.

The notation is the same as before, except that now $N_{s,j,t}$ denotes total labor supplied to firm j in sector s, and $W_{s,j,t}$ is the associated nominal wage rate. $I_{s,j,t}$ denotes investment allocated to firm j in sector s, $I_t \equiv \sum_{s=1}^{S} f_s \int_0^1 I_{s,j,t} dj$, $K_{s,j,t}$ is capital supplied to firm j in sector s, and $Z_{s,j,t}$ is the associated nominal return on capital.

The first-order conditions for consumption and labor are now:

$$\frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} = \frac{\beta^l}{\Theta_{t,l}} \frac{P_t}{P_{t+l}},$$

$$\frac{W_{s,j,t}}{P_t} = N_{s,j,t}^{\gamma} C_t^{\sigma},\ \forall\ s, j.$$

Consumers' allocation of investment and capital to firm j in sector s is such that:

$$Q_{s,j,t} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\frac{Z_{s,j,t+1}}{P_{t+1}} + Q_{s,j,t+1} \left[(1-\delta) + \Phi' \left(\frac{I_{s,j,t+1}}{K_{s,j,t+1}} \right) \left(\frac{I_{s,j,t+1}}{K_{s,j,t+1}} \right)^2 \right] \right) \right\},$$

$$Q_{s,j,t} \left(\Phi' \left(\frac{I_{s,j,t}}{K_{s,j,t}} \right) \frac{I_{s,j,t}}{K_{s,j,t}} + \Phi \left(\frac{I_{s,j,t}}{K_{s,j,t}} \right) \right) = 1,$$

where $Q_{s,j,t}$ denotes Tobin's q of firm j in sector s.

Once we introduce firm-specific factor markets, the intermediate goods producer's problem also changes since wages and the return on capital will also be firm-specific:

$$\begin{array}{ll} \max & E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \left[\begin{array}{c} X_{s,j,t} Y_{s,j,t+l} + \\ -W_{s,j,t+l} N_{s,j,t+l} - Z_{s,j,t+l} K_{s,j,t+l} \end{array} \right] \\ s.t. & \text{(3) and (4).} \end{array}$$

Optimal price setting implies:

$$X_{s,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{s,t+l} \left(\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1}\right)^{-1} W_{s,j,t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{s,t+l}},$$

where $\Lambda_{s,t}$ is defined in equation (4).

From cost-minimization, real marginal costs can be expressed as:

$$MC_{s,j,t} = \frac{1}{\chi^{\chi} \left(1-\chi\right)^{1-\chi}} \left(\frac{W_{s,j,t}}{P_t}\right)^{\chi} \left(\frac{Z_{s,j,t}}{P_t}\right)^{(1-\chi)}. \tag{6}$$

Note that marginal costs are now firm-specific. This is a direct implication of the assumption of firm-specific capital and labor markets.

2.3 Economy-wide factors

Finally, we consider a version of the model in which labor and capital can move freely across firms and sectors. This requires that we reformulate the consumers' and intermediate firms' problems once again. The maximization problem of final goods' firms remains the same as before.

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + P_t I_t + E_t \left[\Theta_{t,t+1} B_{t+1}\right] \leq W_t N_t + B_t + T_t + Z_t K_t,$$

the law of motion for capital stocks:

$$\begin{aligned} K_{t+1} &= \left(1-\delta\right) K_t + \Phi\left(I_t, K_t\right) I_t, \\ I_t &\geq 0, \end{aligned}$$

where $\Phi(I_t, K_t)$ now takes the form:

$$\Phi(I_t, K_t) = \Phi\left(\frac{I_t}{K_t}\right) = 1 - \frac{1}{2}\kappa\frac{\left(\frac{I_t}{K_t} - \delta\right)^2}{\frac{I_t}{K_t}},$$

and a standard "no-Ponzi" condition.

The notation is the same as before, except that N_t is total labor supply, W_t is the corresponding nominal wage rate, I_t denotes investment, K_t stands for capital, Z_t is the associated nominal return on capital.

The first-order conditions for consumption and labor are:

$$\frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} = \frac{\beta^l}{\Theta_{t,l}}\frac{P_t}{P_{t+l}},$$

$$\frac{W_t}{P_t} = N_t^{\gamma} C_t^{\sigma}. \tag{7}$$

Consumers' allocation of investment I_t and capital K_{t+1} yields:

$$Q_t\left(\Phi'\left(\frac{I_t}{K_t}\right)\frac{I_t}{K_t} + \Phi\left(\frac{I_t}{K_t}\right)\right) = 1,$$

$$Q_t = \beta E_t\left\{\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\left(\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1}\left[(1-\delta) + \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right]\right)\right\},$$

where Q_t denotes Tobin's q.

Under economy-wide capital and labor markets, the intermediate firm's problem becomes:

$$\begin{array}{ll} \max & E_t \sum_{l=0}^{\infty} \Theta_{t,t+l}(1-\alpha_s)^l \begin{bmatrix} X_{s,j,t}Y_{s,j,t+l} + \\ -W_{t+l}N_{s,j,t+l} - Z_{t+l}K_{s,j,t+l} \end{bmatrix} \\ s.t. & (3) \text{ and } (4). \end{array}$$

Optimal price setting implies:

$$X_{s,j,t} = \frac{\theta}{\theta-1}\frac{E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}(1-\alpha_s)^l \Lambda_{s,t+l}\left(\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1}\right)^{-1} W_{t+l}}{E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}(1-\alpha_s)^l \Lambda_{s,t+l}},$$

where $\Lambda_{s,t}$ is defined in equation (4).

From cost-minimization, real marginal costs can be expressed as:

$$MC_{s,j,t} = MC_t \frac{1}{\chi^{\chi}(1-\chi)^{1-\chi}}\left(\frac{W_t}{P_t}\right)^{\chi}\left(\frac{Z_t}{P_t}\right)^{(1-\chi)}. \tag{8}$$

Note that marginal costs are equalized across firms and sectors. This is a direct implication of the assumption of economy-wide capital and labor markets.

2.4 Monetary policy

In our baseline specification we assume that the growth rate of nominal aggregate demand follows a first-order autoregressive ($AR(1)$) process, thus leaving monetary policy implicit. This is a common assumption in the literature (e.g. Mankiw and Reis 2002). Denoting nominal aggregate demand by $M_t \equiv P_t Y_t$, we assume:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \sigma_{\varepsilon_m} \varepsilon_{m,t},$$

where $m_t \equiv \log(M_t)$, ρ_m determines the autocorrelation in nominal aggregate demand growth, and $\varepsilon_{m,t}$ is a purely monetary, uncorrelated, zero-mean, unit-variance *i.i.d.* shock.

We analyze the model using a loglinear approximation around the zero-inflation steady state. While solving the models with sectoral and economy-wide factor markets is straightforward, the model with firm-specific capital is more challenging. The reason is that firms can have different capital-accumulation histories. We solve that model by generalizing the approach pioneered by Woodford (2005) to the case of a multisector economy. Details of the solution are available in the Appendix.

3 The underlying New Keynesian Phillips curves

Each model of Section 2 gives rise to a distinct New Keynesian Phillips curve (NKPC), with different implications for aggregate dynamics.[12] As a first step to understand those differences, we can develop some intuition by analyzing the so-called frictionless optimal prices in each model – i.e., the prices that firms would set if they could change them continuously.

For simplicity, we work with versions of the model without capital ($\chi = 1$).[13] In that case, the expressions for the frictionless optimal prices in the economy-wide, sector- and firm-specific economies are given by, respectively:

$$\text{Economy-wide:} \qquad p^*_{j,s,t} = (\sigma + \gamma)\, m_t + (1 - \sigma - \gamma)\, p_t \tag{9}$$

$$\text{Sector-specific:} \qquad p^*_{j,s,t} = (\sigma + \gamma)\, m_t + (1 - \sigma - \gamma + \eta\gamma)\, p_t - \eta\gamma p_{s,t} \tag{10}$$

$$\text{Firm-specific:} \qquad p^*_{j,s,t} = \frac{\sigma + \gamma}{1 + \theta\gamma} m_t + \frac{1 - \sigma - \gamma + \eta\gamma}{1 + \theta\gamma} p_t + \frac{(\theta - \eta)\,\gamma}{1 + \theta\gamma} p_{s,t}, \tag{11}$$

where $p^*_{j,s,t}$ is the frictionless optimal price of firm j in sector s.

The coefficients associated with the aggregate price level (p_t) and with sectoral prices ($p_{s,t}$) indicate how a firm would like to respond to changes in those prices. That is, they determine the nature of

[12] Derivations can be found in the Appendix.

[13] Hence, the analysis in this section is similar to that in Carvalho and Lee (2011).

pricing interactions in the three economies. Note that, because these are multisector economies, it is possible that firms' pricing decisions interact differently, depending on whether firms are in the same sector or in different sectors. We refer to the dependence of $p^*_{j,s,t}$ on $p_{s,t}$ as resulting from *within-sector* pricing interactions, and to the effect of p_t on $p^*_{j,s,t}$ as resulting from *across-sector* pricing interactions.[14]

A positive coefficient on the aggregate price level, say, implies that a firm's frictionless optimal price rises when other prices increase. That is, such a positive coefficient implies a strategic complementarity with respect to the overall level of prices in the economy (i.e., across-sector complementarity). Likewise, a negative coefficient on the sectoral price index implies strategic substitutability with respect to the prices of firms in the same sector (i.e., within-sector substitutability).

In order to think through the patterns of pricing interactions that the three models can deliver, let us start with the case of economy-wide factor markets (equation 9). In that model, in response to, say, an increase in the aggregate price level, a firm j in sector s will have an incentive to increase, decrease, or to keep its price unchanged depending on wether $(1 - \sigma - \gamma)$ is positive, negative, or zero, respectively. So, to fix ideas, let us consider as a benchmark the parameterization under which the economy-wide model features *strategic neutrality* in price setting – i.e., when $\sigma + \gamma = 1$. We can, then, more easily compare the three models.

When $\sigma + \gamma = 1$, both the sector- and the firm-specific models tend to induce strategic complementarity in price setting across sectors. This happens as long as $\eta\gamma > 0$ – that is, as long as there is some substitution between varieties in different sectors $(\eta > 0)$ and the Frisch elasticity of labor supply is finite $(\gamma > 0)$. Note also, that, as long as the latter condition holds $(\gamma > 0)$, the degree of across-sector pricing complementarities in the model with sector-specific labor is larger than in the model with firm-specific labor: $1 - \sigma - \gamma + \eta\gamma \geq (1 - \sigma - \gamma + \eta\gamma) / (1 + \theta\gamma)$.

The two models of labor specificity, however, differ markedly in their implications for the pattern of pricing interactions within sectors (assuming $\eta\gamma > 0$). While the model with sector-specific labor necessarily induces strategic substitutability within sectors, the firm-specific model features within-sector strategic complementarity as long as the elasticity of substitution between varieties within any given sector exceeds the elasticity of substitution between varieties across sectors $(\theta > \eta)$ – which seems plausible.

When $\sigma + \gamma \neq 1$, the pattern of pricing interactions can differ even more across the three economies, depending on parameter values.

Let us now turn to the economic mechanisms underlying these possible patterns of pricing interactions. We rely on the usual thought experiment of considering exogenous changes in aggregate or

[14] Strictly speaking, this separation of the effects of $p_{s,t}$ and p_t on $p^*_{j,s,t}$ only holds exactly if sectors have negligible mass. Nevertheless, we adopt this terminology for expositional convenience.

sectoral prices, while keeping nominal aggregate demand constant.

We start with the economy-wide factors model. In this model, it is clear from (9) that sectoral prices have no effect on a firm's frictionless optimal price. The reason is that, in the labor-only models that we entertain in this section, that price is proportional to the nominal wage. With an economy-wide labor market, the nominal wage only depends on the aggregate price level, aggregate labor, and aggregate consumption. These are unaffected by changes in sectoral prices that keep the aggregate price level constant (which is the right thought experiment to isolate how sectoral prices affect a firm's frictionless optimal price).

Turning to the effects of the aggregate price level on a firm's frictionless optimal price, let us consider an exogenous increase in the overall level of prices. With nominal aggregate demand constant by assumption, consumption falls (in the proportion of the price increase) – and so does labor. Whether the nominal wage ends up increasing, decreasing, or unchanged thus hinges on the Frisch elasticity of labor supply and on the intertemporal elasticity of substitution in consumption (see equation 7). If these elasticities are large (i.e., γ and σ are small), the real wage falls relatively little. That is, the nominal wage – and hence, a firm's frictionless optimal price – moves closely with the aggregate price level. This explains why pricing decisions are strategic complements when the Frisch elasticity of labor supply and/or the intertemporal elasticity of substitution in consumption are large enough (so that $\gamma + \sigma < 1$). The knife-edge case of strategic neutrality ($\gamma + \sigma = 1$) arises precisely when these elasticities are such that the drop in consumption and labor exactly offset the aggregate price increase and keep nominal wages unchanged.

We now take up the case of firm-specific labor. Once again, understanding the effects of changes in aggregate or sectoral prices on a firm's frictionless optimal price requires understanding their effects on nominal wages. The difference is that, now, wages depend on aggregate consumption and firm-level, as opposed to aggregate, employment. As before, let us first consider an exogenous increase in the overall level of prices. With nominal aggregate demand constant by assumption, consumption falls like before (in the proportion of the price increase). But, in general, the level of labor employed by the firm under consideration can increase or decrease. On the one hand, the drop in aggregate demand tends to lower that firm's demand for labor, just as in the economy-wide case. On the other hand, the assumed aggregate price increase makes the firm's variety relatively cheaper. This increases demand for its variety, and hence its demand for labor. For parameter values that imply strategic neutrality in the economy-wide model ($\gamma + \sigma = 1$), we can conclude that the nominal wage ends up increasing. The reason is that, for those parameter values, the drop in aggregate consumption and its negative effect on the firm's labor demand exactly offset the aggregate price increase. So, the net effect on the nominal wage the firm has to pay its workers is positive, and is dictated by the increase in relative demand for its variety.[15]

Continuing with the model with firm-specific labor, let us now analyze the effects of a change in the level of prices in a given sector ($p_{s,t}$) on the frictionless optimal price of a firm in that sector ($p^*_{j,s,t}$). For concreteness, let us assume an increase in $p_{s,t}$. What matters for the effect of such a price change on the nominal wage that firm j has to pay its worker is whether the firm ends up having to employ more or less labor. There are two opposite forces at work. First, the increase in $p_{s,t}$ reduces the demand for varieties of all firms in sector s. This effect is dictated by the elasticity of substitution between varieties across sectors (η). The second effects arises because the assumed sectoral price increase makes firm j's variety relatively cheaper. This increases demand for its variety, and hence its demand for labor. This effect is dictated by the elasticity of substitution between varieties in the same sector (θ). Under the plausible assumption that the elasticity of substitution between varieties within any given sector exceeds the elasticity of substitution between varieties across sectors ($\theta > \eta$), the net effect on firm j's employment, and hence on the nominal wage it has to pay its workers, is positive. Therefore, under this assumption, the firm-specific model features across-sector strategic complementarities in price-setting decisions.

We now turn to the model with sector-specific labor markets. Like before, understanding the effects of changes in aggregate or sectoral prices on a firm's frictionless optimal price requires understanding their effects on nominal wages. The difference is that, now, wages depend on aggregate consumption and employment at the sector – as opposed to aggregate or firm – level. Consider first an exogenous increase in the overall level of prices. With nominal aggregate demand constant by assumption, consumption falls like before (in the proportion of the price increase). But, in general, the level of labor employed in the sector under consideration can increase or decrease. On the one hand, the drop in aggregate demand tends to lower the sector's demand for labor. On the other hand, the assumed aggregate price increase makes all varieties in that sector relatively cheaper. This increases demand for that sector's output, and hence its demand for labor. For parameter values that imply strategic neutrality in the economy-wide model ($\gamma + \sigma = 1$), we can conclude that the nominal wage in the sector at hand increases. The reason is that, for those parameter values, the drop in aggregate consumption and its negative effect on the sector's labor demand exactly offset the aggregate price increase. So, the net effect on the sector's nominal wage is positive, and is dictated by the increase in relative demand for that sector's output.[16]

[15] More generally, a comparison of equations (11) and (9) shows that: i) if the economy-wide model features strategic substitutability in price setting, then the firm-specific model features less substitutability and may even imply across-sector strategic complementarities; ii) if the economy-wide model features strategic complementarity in price setting, then the firm-specific model will also feature across-sector strategic complementarities, although they may be stronger or weaker than in the former model.

[16] Unlike in the case of firm-specific labor markets, we can ascertain that the model with sectoral labor markets features stronger across-sector strategic complementarities (or weaker across-sector strategic substitutability) than the model with an economy-wide labor market.

Finally, we analyze the effects of a change in the level of prices in a given sector ($p_{s,t}$) on the frictionless optimal price of a firm in that sector ($p^*_{j,s,t}$), in the model with sectoral labor markets. For concreteness, let us assume an increase in $p_{s,t}$. Here the effect is straightforward. The increase in $p_{s,t}$ reduces the demand for varieties of all firms in sector s, to an extent dictated by the elasticity of substitution between varieties across sectors (η). This reduces the demand for labor by firms in that sector, which decreases wages to an extent dictated by the inverse of the Frisch elasticity of labor supply (γ). Therefore the model with sector-specific labor markets unequivocally induces within-sector strategic substitutability in price-setting decisions.

3.1 Phillips curves

We start with the more familiar case of economy-wide capital and labor markets. It leads to the following NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + (\bar{\sigma} + \omega)\left(\sum_{s=1}^{S} f_s \mu_s\right)\tilde{y}_t + \eta^{-1}\sum_{s=1}^{S} f_s \mu_s \left(\tilde{y}_{s,t} - \tilde{y}_t\right) - \bar{\sigma}\left(\sum_{s=1}^{S} f_s \mu_s\right)\tilde{\imath}_t, \tag{12}$$

where $\omega = (\gamma + 1 - \chi)/\chi$, $\bar{\sigma} = (Y/C)\,\sigma$, $\mu_s = \alpha_s (1 - \beta (1 - \alpha_s))/(1 - \alpha_s)$, and variables with a tilde superscript denote deviations from the underlying flexible-price equilibrium.[17] The last term on the right-hand-side of (12) is due to endogenous capital accumulation,[18] and the third term is reminiscent of multisector models with heterogeneity in price rigidity (Carvalho 2006). The second term is the standard output gap component of the NKPC.[19]

Note that in a model without capital and with the same frequency of price changes in all sectors (α, say), equation (12) collapses to the standard NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + (\sigma + \omega)\,\mu \tilde{y}_t,$$

where $\mu = \alpha (1 - \beta (1 - \alpha))/(1 - \alpha)$. The coefficient $\bar{\sigma} + \omega$ that multiplies the output gap term in (12) is thus seen to correspond to the Ball and Romer (1990) index of real rigidities.[20]

Turning to the model with firm-specific factors (Section 2.2), the underlying NKPC is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \bar{\sigma}\left(\sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s}\right)\tilde{y}_t + \omega \sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s}\tilde{y}_{s,t} + \eta^{-1}\sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s}\left(\tilde{y}_{s,t} - \tilde{y}_t\right) - \bar{\sigma}\left(\sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s}\right)\tilde{\imath}_t, \tag{13}$$

[17] The flexible-price equilibrium is defined as in Woodford (2003, 2005), i.e., it assumes flexible prices going forward, given the current capital stock.

[18] Investment is defined as percentage deviation from its steady state.

[19] Due to heterogeneity in price stickiness, the coefficient associated with nominal price rigidities is a weighted average of the associated sectoral coefficients $\left(\sum_{s=1}^{S} f_s \mu_s\right)$.

[20] For a thorough discussion of sources of real rigidities in the canonical New Keynesian model see Woodford (2003, Ch. 3).

where ϕ_s is a function of structural parameters that is obtained (numerically) as the solution of a system of nonlinear equations (see the Appendix).

Equation (13) generalizes the NKPC derived in Woodford (2005) to the case of a multisector economy with heterogeneity in price rigidity. The main difference relative to the case of economy-wide factor markets is that all "gaps" are now multiplied by ϕ_s^{-1}. As in Woodford (2005), it is the case that $\phi_s > 1$ under reasonable parameterizations. Thus, under those parameterizations, firm-specific factors are seen to mute the sensitivity of inflation to aggregate and sectoral output gaps, and the investment gap. Firm-specificity is thus a source of real rigidities in the Ball-Romer sense. This is most easily seen in a version of this model without capital and with the same frequency of price changes in all sectors. In that case the NKPC simplifies to:

$$\pi_t = \beta E_t \pi_{t+1} + (\sigma + \omega) \frac{\mu}{\phi} \tilde{y}_t, \tag{14}$$

where $\phi = 1 + \theta\gamma$.

When capital and labor are sector-specific, the NKPC becomes:

$$\pi_t = \beta E_t \pi_{t+1} + \bar{\sigma} \left(\sum_{s=1}^{S} f_s \mu_s \right) \tilde{y}_t + \omega \sum_{s=1}^{S} f_s \mu_s \tilde{y}_{s,t} + \eta^{-1} \sum_{s=1}^{S} f_s \mu_s \left(\tilde{y}_{s,t} - \tilde{y}_t \right) - \bar{\sigma} \left(\sum_{s=1}^{S} f_s \mu_s \right) \tilde{\imath}_t. \tag{15}$$

Note that this Phillips curve is almost identical to the one that obtains under firm-specific factor markets (equation 13), with the crucial exception that there are no ϕ_s^{-1} coefficients multiplying the output and investment gaps. The reason is that factor prices are now equalized within sectors, so that, in contrast to the model with firm-specific factors, individual firms' pricing decisions have no impact on their marginal cost.

Note that, if sectors are identical in terms of price rigidity, the dynamics of sectoral output gaps in response to an aggregate disturbance will be the same in all sectors ($\tilde{y}_{s,t} = \tilde{y}_t$), and equations (12) and (15) become identical, and simplify to:

$$\pi_t = \beta E_t \pi_{t+1} + (\bar{\sigma} + \omega) \mu \tilde{y}_t - \bar{\sigma} \mu \tilde{\imath}_t.$$

The same does not happen when factors are firm-specific, because firms still need to internalize the effects of their pricing decisions on their marginal cost, through factor prices. The corresponding equation with homogeneous price rigidity simplifies to:

$$\pi_t = \beta E_t \pi_{t+1} + (\bar{\sigma} + \omega) \frac{\mu}{\phi} \tilde{y}_t - \bar{\sigma} \frac{\mu}{\phi} \tilde{\imath}_t,$$

where the ϕ coefficient embeds the effects of firm specificity.[21]

[21] Here we abuse notation because, due to capital accumulation, ϕ does not equal that in equation (14).

When sectors are heterogeneous, the Phillips curve with sectoral specificities differs from the Phillips curve under economy-wide factor markets only in that ω now multiplies sectoral output gaps rather than the aggregate output gap. The reason for this difference is that factor prices now also depend on sectoral conditions. The $\omega \sum_{s=1}^{S} f_s \mu_s \tilde{y}_{s,t}$ term summarizes the effects of sectoral conditions on marginal costs, keeping aggregate output constant. These effects operate through increasing marginal disutility of labor and decreasing marginal product of capital – which affect sectoral factor prices. The $\bar{\sigma} \left(\sum_{s=1}^{S} f_s \mu_s \right) \tilde{y}_t$ term captures the effects of aggregate conditions on marginal costs, which continue to operate through households' decreasing marginal utility of consumption. In the model with economy-wide factor markets, the first effect hinges on the aggregate output gap, rather than on sectoral gaps.[22]

4 Quantitative analysis

In this section, we parameterize our three models and analyze their quantitative predictions. We set the intertemporal elasticity of substitution σ^{-1} to 1/2, the (inverse) labor supply elasticity γ to 0.5, and the elasticity of output with respect to labor to $\chi = 2/3$. The consumer discount factor β implies a time-discount rate of 4% per year.

We set the elasticity of substitution between varieties of the same sector to $\theta = 7$. The elasticity of substitution between varieties of different sectors should arguably be smaller than within sectors. We assume a unit elasticity of substitution across sectors, $\eta = 1$ (i.e. the aggregator that converts sectoral into final output is Cobb-Douglas).

To specify the process for nominal aggregate demand, the literature usually relies on estimates based on nominal GDP, or on monetary aggregates such as M1 or M2. With quarterly data, estimates of ρ_m typically fall in the range of 0.4 to 0.7,[23] which maps into a range of roughly $0.75 - 0.90$ at a monthly frequency. We set $\rho_m = 0.8$, and the standard deviation of the shocks $\sigma_{\varepsilon_m} = 0.6\%$ (roughly 1% at a quarterly frequency), in line with the same estimation results.[24]

To calibrate investment adjustment costs, we follow an approach that is common in the real business cycle literature (e.g., Chari et al. 2000) and calibrate the investment adjustment-cost parameter (κ) to match the standard deviation of investment in the data relative to the standard deviation of GDP. Whenever we analyze a different version of the model, we redo the calibration.[25]

It remains to specify the distribution of price rigidity. We start by investigating whether the

[22] Incidentally, note that in equation (13) ω also multiplies sectoral output gaps. The reason is that the solution of the model is such that the effects of firm specificity are subsumed in ϕ_s, and the model is in effect solved by taking sectoral averages (see the Appendix).

[23] See, for instance, Mankiw and Reis (2002).

[24] All results for volatilities scale-up proportionately with σ_{ε_m}.

[25] It turns out that the calibrations of the three versions of the model yield very similar values for the investment adjustment-cost parameter ($\kappa \approx 40$).

model with sectoral factor markets is flexible enough to approximate both the model with firm-specific factors and the model with economy-wide factor markets. To that end, we experiment with different arbitrary distributions of price stickiness in economies with 3 sectors.[26] Subsequently, we use the available microeconomic evidence on price rigidity in the U.S. to pin down the distribution of price rigidity across sectors.

4.1 Arbitrary distributions of price rigidity

We analyze economies with 3 sectors, and entertain two alternative distributions of price rigidity. In both cases the frequencies of price changes (α_s) in sectors 1-3 are, respectively, 1, 1/12, and 1/30 – corresponding to price changes of, on average, once a month, once a year, and once every 30 months. We differentiate the two distributions by changing the sectoral weights.

In the first distribution, which we term "flexible distribution," the weights of sectors 1-3 are, respectively, 85%, 7.5%, and 7.5%. In the second distribution, which we label "sticky distribution," the weights of sectors 1-3 are, respectively, 7.5%, 7.5%, and 85%.

Figure 1 presents the impulse response functions of real GDP for the two alternative sets of sectoral weights. The panel on the left shows the impulse response functions of GDP under the flexible distribution for the three alternative models of factor markets, while the panel on the right shows the responses for the three models under the sticky distribution. Note that the scales on the two charts are different.

The results show that, under the flexible distribution, the sector-specific factor market economy generates more monetary non-neutrality than the other two models. In contrast, under the sticky distribution that model is more similar to the model with economy-wide factor markets. This result suggests that, depending on the distribution of price rigidity, the pattern of pricing interactions reflected in the NKPC under sectoral factor markets is rich enough to emulate the dynamics of both an economy with strong real rigidities (such as the one with firm-specific factor markets) and an economy with strategic substitutability in pricing decisions (such as the one with economy-wide factor markets).

4.2 Empirical distribution of price rigidity

As the previous section shows, the model with sectoral factor markets is rich enough to generate more or less monetary non-neutrality than the other two models that we consider, depending on the distribution of price rigidity. We now restrict our quantitative analysis to versions of the models

[26] We do so motivated by our finding that a 3-sector economy with a suitably chosen distribution of price rigidity provides a very good approximation to the dynamics of a multisector economy calibrated to the empirical distribution of price rigidity in the U.S. See details below.

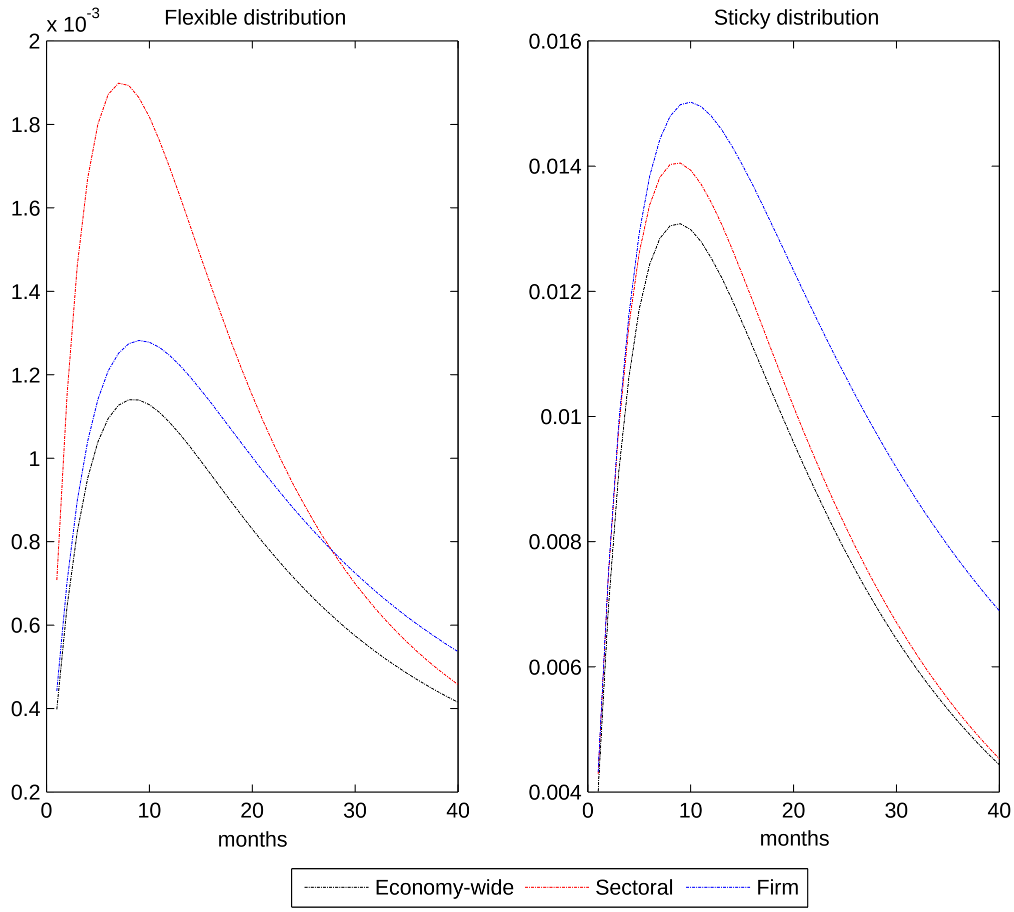

Figure 1: IRFs of GDP to monetary shocks – arbitrary distributions of price rigidity

calibrated with the empirical distribution of price rigidity for the U.S. economy. We use the statistics on the frequency of regular price changes – those that are not due to sales or product substitutions – reported by Nakamura and Steinsson (2008). To make the model computationally manageable, we build from the statistics for 271 categories of goods and services,[27] and aggregate those 271 categories into 67 expenditure classes. Each class is identified with a sector in the model.[28] The frequency of price changes for each expenditure class is obtained as the weighted average of the frequencies for the underlying categories, using the expenditure weights provided by Nakamura and Steinsson (2008). Finally, expenditure-class weights are given by the sum of the expenditure weights for those categories. The resulting average monthly frequency of price changes is $\overline{\alpha} = \sum_{k=1}^{K} f_k \alpha_k = 0.211$, which implies that prices change on average once every 4.7 months.

Solving, calibrating and simulating the multisector model with 67 sectors is computationally costly. To sidestep this problem we work with a 3-sector approximation to the underlying 67-sector economy. We choose the frequencies of price changes and sectoral weights in the approximating model to match a suitably chosen set of moments of the cross-sectional distribution of price stickiness of the original 67-sector economy. This delivers an extremely good approximation to the dynamics of a 67-sector model, under all three assumptions about factor markets. See the Appendix for details.

Figure 2 reports our main results. It shows impulse response functions to a monetary shock for the three models, economy-wide, sectoral, and firm-specific factor markets. Focusing first on the IRFs for GDP, the chart shows that the economy-wide factor model implies the smallest monetary non-neutrality. The sectoral and firm-specific factor models are more similar on impact and during the first few months, featuring more sizable non-neutralities than the economy-wide model. In the medium to long run, however, the model with sectoral specificity quickly approaches the one with economy-wide factor markets. At the end of the day, the model with firm-specific factors generates longer lasting effects of monetary shocks on real GDP than the other two models.

The response of inflation in the three models displays a similar pattern. The short-run response of the model with sector-specific factors resembles that of model with firm-specific factors. Over time, the economy with sector specificity approaches the behavior of the model with economy-wide factor markets. Loosely speaking, if one analyzes the joint behavior of inflation and output in response to a monetary shock, the economy with sector-specific factor markets behaves "as if" it featured a Phillips curve with a slope that is larger at higher frequencies than at lower frequencies – when compared with

[27] Nakamura and Steinsson (2008) report statistics for 272 categories. We discard the category "Girls' Outerwear", for which the reported frequency of regular price changes is zero. We renormalize the expenditure weights to sum to unity.

[28] As an example of what this aggregation entails, the resulting "New and Used Motor Vehicles" class consists of the categories "Subcompact Cars", "New Motorcycles", "Used Cars", "Vehicle Leasing" and "Automobile Rental"; the "Fresh Fruits" class comprises four categories: "Apples", "Bananas", "Oranges, Mandarins etc." and "Other Fresh Fruits."

the other two economies.[29] The remaining charts in Figure 2 report the impulse response functions for other main variables in the three models.

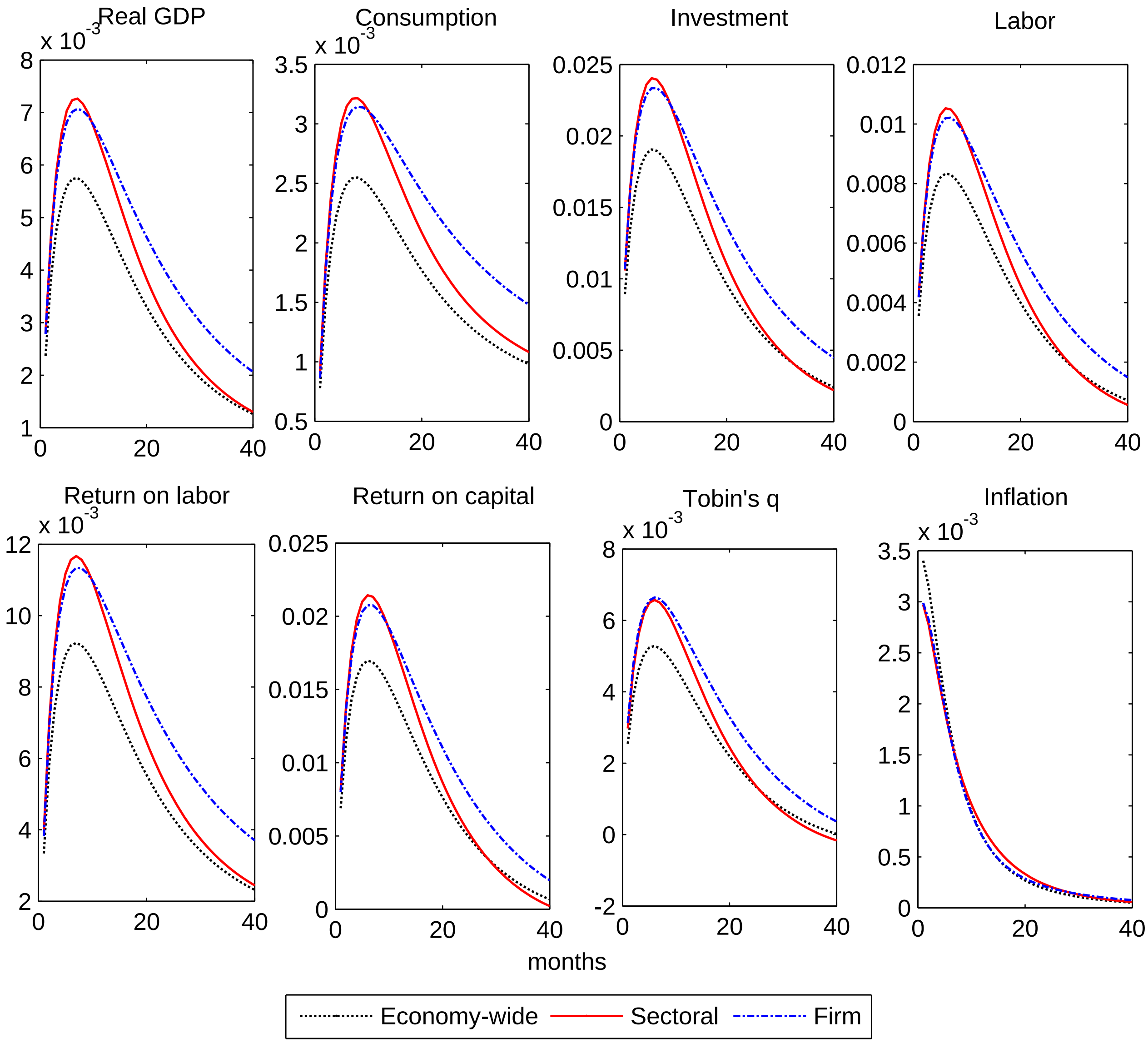

Figure 2: Monetary shocks – impulse response functions

4.3 Robustness

In our previous analyses we postulated an exogenous stochastic process for nominal aggregate demand. In this section, we consider a specification with an explicit description of monetary policy. We assume

[29] We thank Jeff Campbell for this insight.

that policy is conducted according to an interest-rate rule subject to persistent shocks:

$$IR_t = \beta \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{GDP_t}{GDP} \right)^{\phi_Y} e^{\upsilon_t},$$

where IR_t is the short-term nominal interest rate, GDP_t is gross domestic product, GDP denotes gross domestic product in steady state, ϕ_π and ϕ_Y are the parameters associated with Taylor-type interest-rate rules (Taylor 1993), and υ_t is a persistent shock with process $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \sigma_{\varepsilon_\upsilon} \varepsilon_{\upsilon,t}$, where $\varepsilon_{\upsilon,t}$ is a zero-mean, unit-variance *i.i.d.* shock, and $\rho_\upsilon \in [0, 1)$. We set $\phi_\pi = 1.5$, $\phi_y = .5/12$, and $\rho_\upsilon = 0.965$.[30] The remaining parameter values are unchanged from the baseline specification.

Figure 3 shows that the main findings reported in the previous section are robust to modeling monetary policy with an explicit interest rate rule.[31] In particular, focusing on the impulse response functions for real GDP, the sector-specific factor model initially resembles the response of the firm-specific factor model. Over time, its adjustment speeds up and the impulse response function approaches that of the model with economy-wide factor markets. The real effects of monetary shocks in the model with firm-specific factors are not only larger but also more persistent. The same patterns hold for the remaining variables depicted in Figure 3.

5 Conclusion

Our results show that it matters a great deal whether specificity in factors of production arises at the firm or at the sectoral level. In response to nominal shocks, our parameterized model with sector-specific factor markets yields aggregate dynamics in the short run that resemble those of a model with firm-specific factors. After one to two years, however, the behavior of the model is already more similar to that of a model with economy-wide factor markets.

This result is consistent with the idea that factor price equalization might take place gradually over time, so that firm-specificity might be a reasonable short-run approximation, whereas economy-wide markets might be a better description of how factors of production are allocated in the longer run. Whether or not this happens in reality is, of course, an empirical question. Existing empirical evidence that both capital (e.g., Ramey and Shapiro 2001) and labor (e.g., Parent 2000) have some degree of sector (or industry) specificity suggests that our model may imply plausible aggregate dynamics.

[30] Recall that the parameters are calibrated to the monthly frequency, and so this value for ρ_υ corresponds to an autoregressive coefficient of roughly 0.9 at a quarterly frequency. We specify the size of the shocks to be consistent with the estimates of Justiniano et al. (2010), and thus set the standard deviation to 0.2% at a quarterly frequency.

[31] Our findings about the implications of the model with sector-specific factor markets are robust to using a monetary policy rule with interest rate smoothing. The main difference relative to the results reported in Figure 3 is that the overall level of persistence of real variables in response to the monetary shock is substantially lower (a result that is consistent with the findings of Carvalho and Nechio 2014).

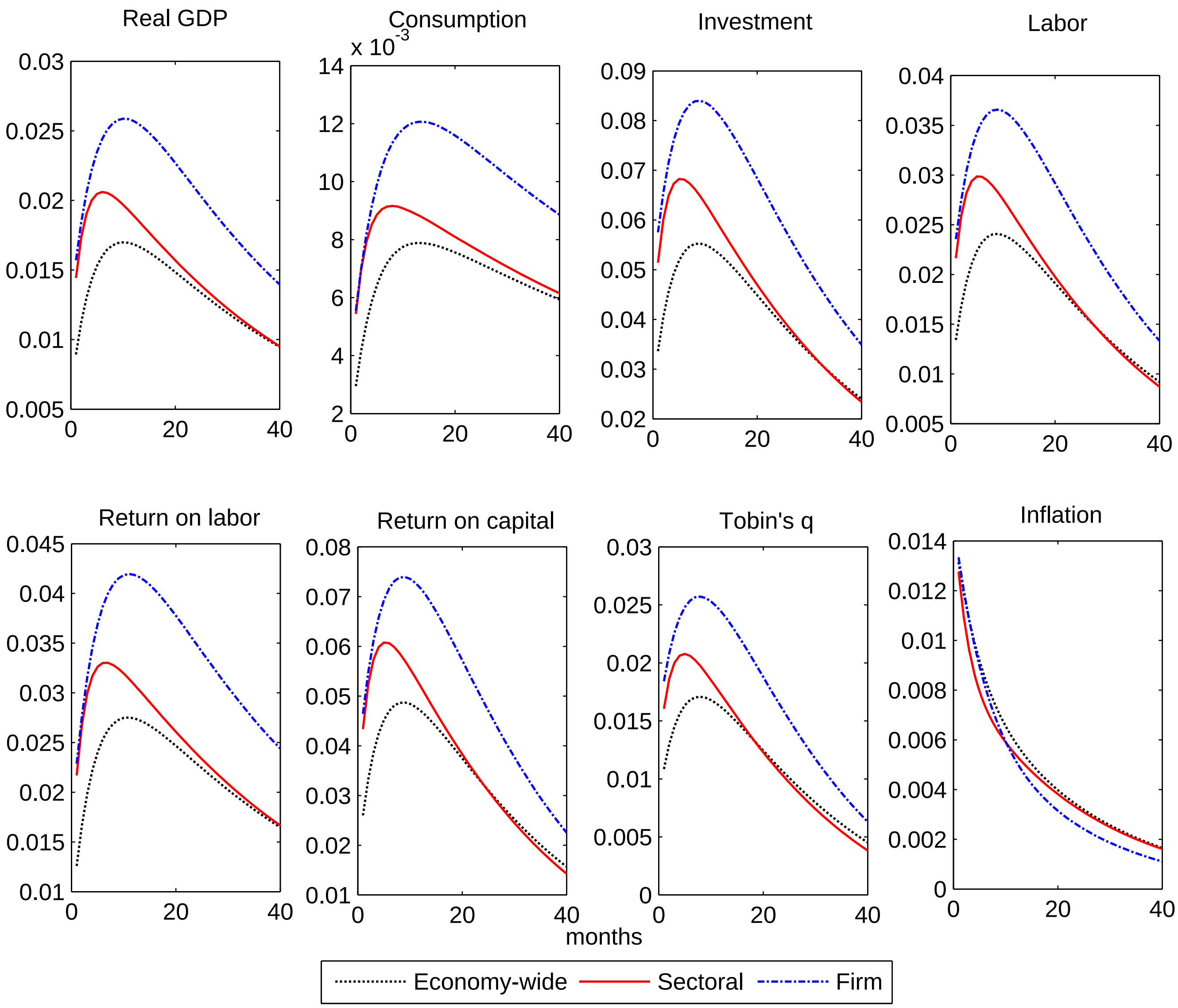

Figure 3: Monetary shocks under a Taylor rule – impulse response functions

Finally, one implication of our parameterized model is that it behaves "as if" it featured a Phillips curve with a slope that is larger at higher frequencies than at lower frequencies – when compared with the other two economies. It might be possible to test if the data support this implication.

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A Appendix

A.1 Firm-specific model solution

To solve the model, we follow Woodford (2005) and generalize his solution method for a multisector economy.

A.1.1 Rewriting the marginal cost equation

The loglinear versions of the marginal cost, Euler equation, and the production function are given by, respectively:[32]

$$
\begin{aligned}
mc_{s,j,t} &= w_{s,j,t} - p_t - (1-\chi)\, k^d_{s,j,t} - (\chi - 1)\, n_{s,j,t}, && (16)\\
w_{s,j,t} - p_t &= \sigma c_t + \gamma n_{s,j,t}, && (17)\\
y_{s,j,t} &= (1-\chi)\, k_{s,j,t} + \chi n^d_{s,j,t}. && (18)
\end{aligned}
$$

Equations (16)-(18) yield:

$$
k_{s,j,t} = n_{s,j,t} + (w_{s,j,t} - p_t) - (z_{s,j,t} - p_t)\,. \tag{19}
$$

The marginal cost can be rewritten as a function of capital and labor by replacing (19) into (18), isolating for $n_{s,j,t}$, and replacing for equation (17) and (18) to obtain:

$$
\begin{aligned}
mc_{s,j,t} &= w_{s,j,t} - p_t - (1-\chi)\, k_{s,j,t} - (\chi - 1)\, n_{s,j,t}\\
&= \sigma c_t + \gamma n_{s,j,t} - (1-\chi)\, k_{s,j,t} - (\chi - 1)\, n_{s,j,t}. && (20)
\end{aligned}
$$

From the production function (18):

$$
\begin{aligned}
y_{s,j,t} &= (1-\chi)\, k_{s,j,t} + \chi n_{s,j,t}\\
&\Rightarrow \; n_{s,j,t} = \frac{1}{\chi}\left[y_{s,j,t} - a_t - (1-\chi)\, k_{s,j,t}\right].
\end{aligned}
$$

Substituting for $n_{s,j,t}$ in equation (20) and rearranging yields:

$$
mc_{s,j,t} = \sigma c_t + \left(\frac{\gamma - (\chi - 1)}{\chi}\right) y_{s,j,t} - \frac{(1-\chi)\,(1+\gamma)}{\chi} k_{s,j,t}. \tag{21}
$$

[32] Lower-case variables are loglinear deviations from steady state of their upper-case counterparts.

Integrating across all firms in sector s:

$$\begin{aligned} mc_{s,t} &= \sigma c_t + \left(\frac{\gamma - (\chi - 1)}{\chi}\right) y_{s,t} - \frac{(1-\chi)(1+\gamma)}{\chi} k_{s,t} \\ &\Rightarrow \sigma c_t = mc_{s,t} - \left(\frac{\gamma - (\chi - 1)}{\chi}\right) y_{s,t} + \frac{(1-\chi)(1+\gamma)}{\chi} k_{s,t}, \end{aligned} \tag{22}$$

which we can use to replace at equation (21):

$$\begin{aligned} mc_{s,j,t} &= \sigma c_t + \left(\frac{\gamma - (\chi - 1)}{\chi}\right) y_{s,j,t} - \frac{(1-\chi)(1+\gamma)}{\chi} k_{s,j,t} \\ &= mc_{s,t} + \frac{\gamma - (\chi - 1)}{\chi} (y_{s,j,t} - y_{s,t}) - \frac{(1-\chi)(1+\gamma)}{\chi} (k_{s,j,t} - k_{s,t}). \end{aligned}$$

The final firm's maximization problem yields demand for goods produced by firm j in sector s:

$$y_{s,j,t} = y_{s,t} - \theta [p_{s,j,t} - p_{s,t}],$$

which can be used to substitute for $(y_{s,j,t} - y_{s,t})$, to obtain:

$$mc_{s,j,t} = mc_{s,t} - \theta \frac{\gamma - (\chi - 1)}{\chi} (p_{s,j,t} - p_{s,t}) - \frac{(1-\chi)(1+\gamma)}{\chi} (k_{s,j,t} - k_{s,t}). \tag{23}$$

A.1.2 Capital equation

From the consumers' maximization problem, capital and investment allocations are:

$$q_{s,j,t} = E_t \left\{ \begin{array}{c} -\sigma (c_{t+1} - c_t) + [1 - (1-\delta)\beta] (z_{s,j,t+1} - p_{t+1}) \\ + (1-\delta) \beta q_{s,j,t+1} + \kappa \delta^2 \beta (i_{s,j,t+1} - k_{s,j,t+1}) \end{array} \right\},$$

$$q_{s,j,t} = \kappa \delta (i_{s,j,t} - k_{s,j,t}).$$

From these two equations, we get:

$$\kappa \delta (i_{s,j,t} - k_{s,j,t}) = E_t \left\{ \begin{array}{c} -\sigma (c_{t+1} - c_t) + [1 - (1-\delta)\beta] (z_{s,j,t+1} - p_{t+1}) \\ + (1-\delta) \beta \kappa \delta (i_{s,j,t+1} - k_{s,j,t+1}) + \kappa \delta^2 \beta (i_{s,j,t+1} - k_{s,j,t+1}) \end{array} \right\} \tag{24}$$

Using the law of motion for capital stocks:

$$k_{s,j,t+1} - k_{s,j,t} = \delta (i_{s,j,t} - k_{s,j,t}),$$

we can rewrite equation (24) to obtain:

$$\kappa (k_{s,j,t+1} - k_{s,j,t}) = E_t \left\{ \begin{array}{c} -\sigma (c_{t+1} - c_t) + [1 - (1-\delta)\beta] (z_{s,j,t+1} - p_{t+1}) \\ + \kappa \beta (k_{s,j,t+2} - k_{s,j,t+1}) \end{array} \right\}$$

We still need to replace for $z_{s,j,t+1}$ in the above equation. For that we use equations (17)-(18) to obtain:

$$\kappa\left(k_{s,j,t+1}-k_{s,j,t}\right)=E_t\left\{\begin{array}{c}-\sigma\left(c_{t+1}-c_t\right)+\left[1-(1-\delta)\beta\right]\left[\begin{array}{c}\frac{1+\gamma}{\chi}\left(\begin{array}{c}y_{s,j,t+1}\\-(1-\chi)\,k_{s,j,t+1}\end{array}\right)\\+\sigma c_{t+1}-k_{s,j,t+1}\end{array}\right]\\+\kappa\beta\left(k_{s,j,t+2}-k_{s,j,t+1}\right)\end{array}\right\}$$

Integrating across all firms in sector s yields:

$$\kappa\left(k_{s,t+1}-k_{s,t}\right)=E_t\left\{\begin{array}{c}-\sigma\left(c_{t+1}-c_t\right)+\left[1-(1-\delta)\beta\right]\left[\begin{array}{c}\frac{1+\gamma}{\chi}\left(\begin{array}{c}y_{s,t+1}\\-(1-\chi)\,k_{s,t+1}\end{array}\right)\\+\sigma c_{t+1}-k_{s,t+1}\end{array}\right]\\+\kappa\beta\left(k_{s,t+2}-k_{s,t+1}\right)\end{array}\right\}$$

And we can rewrite the equation for capital-investment decision as:

$$\kappa\left[k_{s,j,t+1}-k_{s,t+1}-(k_{s,j,t}-k_{s,t})\right]=E_t\left\{\begin{array}{c}\left[1-(1-\delta)\beta\right]\left[\begin{array}{c}\frac{1+\gamma}{\chi}\left(\begin{array}{c}(y_{s,j,t+1}-y_{s,t+1})\\-(1-\chi)\,(k_{s,j,t+1}-k_{s,t+1})\end{array}\right)\\-(k_{s,j,t+1}-k_{s,t+1})\end{array}\right]\\+\kappa\beta\left[k_{s,j,t+2}-k_{s,t+2}-(k_{s,j,t+1}-k_{s,t+1})\right]\end{array}\right\}$$

Using again the final-firms demand for goods produced by firm j in sector s to substitute for $(y_{s,j,t}-y_{s,t})$, yields:

$$\kappa\left[k_{s,j,t+1}-k_{s,t+1}-(k_{s,j,t}-k_{s,t})\right]=E_t\left\{\begin{array}{c}\left[1-(1-\delta)\beta\right]\left[\begin{array}{c}\frac{1+\gamma}{\chi}\left(\begin{array}{c}-\theta\,(p_{s,j,t+1}-p_{s,t+1})\\-(1-\chi)\,(k_{s,j,t+1}-k_{s,t+1})\end{array}\right)\\-(k_{s,j,t+1}-k_{s,t+1})\end{array}\right]\\+\kappa\beta\left[k_{s,j,t+2}-k_{s,t+2}-(k_{s,j,t+1}-k_{s,t+1})\right]\end{array}\right\}$$

Defining

$$\tilde{p}_{s,j,t}=p_{s,j,t}-p_{s,t}, \tag{25}$$

$$\tilde{k}_{s,j,t}=k_{s,j,t}-k_{s,t}, \tag{26}$$

we can rewrite the previous equation as:

$$\kappa\left[\tilde{k}_{s,j,t+1}-\tilde{k}_{s,j,t}\right]=E_t\left\{\begin{array}{c}\left[1-(1-\delta)\beta\right]\left[\begin{array}{c}\frac{1+\gamma}{\chi}\left(\begin{array}{c}-\theta\tilde{p}_{s,j,t+1}\\-(1-\chi)\,\tilde{k}_{s,j,t+1}\end{array}\right)\\-\tilde{k}_{s,j,t+1}\end{array}\right]\\+\kappa\beta\left[\tilde{k}_{s,j,t+2}-\tilde{k}_{s,j,t+1}\right]\end{array}\right\} \tag{27}$$

A.1.3 Pricing rule

The intermediate-firm optimization problem yields the optimal price setting equation:

$$x_{s,j,t} = (1-\beta(1-\alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m (p_{t+m} + mc_{s,j,t+m})$$

Note that because each firm can have a different capital-accumulation history, expectations may vary from one form to another, and hence, E_t, the usual expectations at time-t operator, differs from E_t^j.

Rewriting equation (23) at $t+m$, and using equations (25) and (26) implies:

$$mc_{s,j,t+m} = mc_{s,t+m} - \theta \frac{\gamma - (\chi - 1)}{\chi} \tilde{p}_{s,j,t+m} - \frac{(1-\chi)(1+\gamma)}{\chi} \tilde{k}_{s,j,t+m},$$

which can be replaced in the optimal pricing equation above to yield:

$$\begin{aligned}
x_{s,j,t} &= (1-\beta(1-\alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m (p_{t+m} + mc_{s,j,t+m}) \\
&= (1-\beta(1-\alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \begin{pmatrix} p_{t+m} + mc_{s,t+m} \\ -\theta \frac{\gamma-(\chi-1)}{\chi} [\tilde{p}_{s,j,t+m}] - \frac{(1-\chi)(1+\gamma)}{\chi} \tilde{k}_{s,j,t+m} \end{pmatrix} \\
&= (1-\beta(1-\alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \begin{pmatrix} p_{t+m} + mc_{s,t+m} \\ -\theta \frac{\gamma-(\chi-1)}{\chi} [\tilde{x}_{s,j,t} - \Sigma_{i=1}^m E_t \pi_{s,t+i}] - \frac{(1-\chi)(1+\gamma)}{\chi} \tilde{k}_{s,j,t+m} \end{pmatrix},
\end{aligned}$$

where we used the fact that definitions (25) and (26) imply:

$$\begin{aligned}
E_t^j \tilde{p}_{s,j,t+m} &= \tilde{p}_{s,j,t} - \Sigma_{i=1}^m E_t \pi_{s,t+i} \\
\tilde{k}_{s,j,t+m} &= k_{s,j,t+m} - k_{s,t+m}
\end{aligned}$$

Taking $p_{s,t}$ from both sides of the price equation above and rewriting:

$$\begin{aligned}
\tilde{x}_{s,j,t} &= (1-\beta(1-\alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \begin{pmatrix} p_{t+m} + mc_{s,t+m} - p_{s,t} \\ -\theta \frac{\gamma-(\chi-1)}{\chi} [\tilde{x}_{s,j,t} - \Sigma_{i=1}^m E_t \pi_{s,t+i}] - \frac{(1-\chi)(1+\gamma)}{\chi} \tilde{k}_{s,j,t+m} \end{pmatrix} \\
&\Rightarrow
\end{aligned}$$

$$\left(1 + \frac{\theta(\gamma - (\chi - 1))}{\chi}\right) \tilde{x}_{s,j,t} = (1-\beta(1-\alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \begin{pmatrix} mc_{s,t+m} + p_{t+m} - [p_{s,t}] \\ +\theta \frac{\gamma-(\chi-1)}{\chi} \Sigma_{i=1}^m E_t \pi_{s,t+i} \\ - \frac{(1-\chi)(1+\gamma)}{\chi} \tilde{k}_{s,j,t+m}. \end{pmatrix}$$

Using the fact that $p_{s,t} = p_{s,t+m} - \Sigma_{i=1}^{m} E_t \pi_{s,t+i}$, we can rewrite:

$$\left(1 + \frac{\theta\left(\gamma - (\chi - 1)\right)}{\chi}\right) \tilde{x}_{s,j,t} = \left(1 - \beta\left(1 - \alpha_s\right)\right) E_t^j \sum_{m=0}^{\infty} \beta^m \left(1 - \alpha_s\right)^m \left(\begin{array}{c} mc_{s,t+m} + p_{t+m} - p_{s,t+m} \\ + \left[1 + \theta \frac{\gamma - (\chi - 1)}{\chi}\right] \Sigma_{i=1}^{m} E_t \pi_{s,t+i} \\ - \frac{(1-\chi)(1+\gamma)}{\chi} \tilde{k}_{s,j,t+m}. \end{array} \right)$$

Note that the only term in the above expression that depends on firm-j expectations is the one associated with capital stocks. Hence, we can rewrite the above expression as:

$$\begin{aligned} \left(1 + \theta \frac{\gamma - (\chi - 1)}{\chi}\right) \tilde{x}_{s,j,t} \quad &= \quad \left(1 - \beta\left(1 - \alpha_s\right)\right) E_t \sum_{m=0}^{\infty} \beta^m \left(1 - \alpha_s\right)^m \left(\begin{array}{c} p_{t+m} + mc_{s,t+m} - p_{s,t+m} \\ + \left(1 + \theta \frac{\gamma - (\chi - 1)}{\chi}\right) \Sigma_{i=1}^{m} E_t \pi_{s,t+i} \end{array} \right) \\ &\quad - \frac{\left(1 - \chi\right)\left(1 + \gamma\right)}{\chi} \left(1 - \beta\left(1 - \alpha_s\right)\right) E_t^j \sum_{m=0}^{\infty} \beta^m \left(1 - \alpha_s\right)^m \tilde{k}_{s,j,t+m}. \end{aligned} \tag{28}$$

A.1.4 Guessing a solution

Equation (27) can be rewritten as:

$$0 = E_t \left\{ \begin{array}{c} \beta \tilde{k}_{s,j,t+2} - \left[\beta + \left[1 - (1 - \delta)\beta\right] \left[\frac{\chi + (1+\gamma)(1-\chi)}{\kappa\chi}\right] - 1\right] \tilde{k}_{s,j,t+1} + \tilde{k}_{s,j,t} \\ - \left[1 - (1 - \delta)\beta\right] \frac{\theta(1+\gamma)}{\kappa\chi} \tilde{p}_{s,j,t+1} \end{array} \right\}$$

$$\begin{aligned} E_t \left\{ Q\left(L\right) \tilde{k}_{s,j,t+2} \right\} \quad &= \quad \Xi_1 E_t \tilde{p}_{s,j,t+1} \\ Q\left(L\right) \quad &= \quad \beta - AL + L^2 \\ \text{where } A \quad &= \quad \left[\beta + \left[1 - (1 - \delta)\beta\right] \left[\frac{\chi + \left(1 + \gamma\right)\left(1 - \chi\right)}{\kappa\chi}\right] - 1\right] \\ \Xi \quad &= \quad \left[1 - (1 - \delta)\beta\right] \frac{\theta\left(1 + \gamma\right)}{\kappa\chi} \end{aligned} \tag{29}$$

And we could factor the $Q(L)$ so to obtain two real roots (see Woodford (2005)).

Because consumer j's decision problem is locally convex, the first order condition characterizes a locally unique optimal plan, and at the time of price adjustment, the chosen price must depend only on j's relative capital stock and its own sector's state. Hence, j's pricing decision must take the form:

$$\tilde{x}_{s,j,t} = g_{s,t} - \psi_s \tilde{k}_{s,j,t}, \tag{30}$$

where $g_{s,t}$ depends only on the sectoral state and aggregate variables, and the coefficient ψ_s is to be determined.

Sectoral prices are such that:

$$P_{s,t} = \left[\int_0^1 P_{s,j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} = \left[\int_I X_{s,i,t}^{1-\theta} di + \int_J P_{s,j,t-1}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}.$$

Dividing both sides by $P_{s,t}$ and loglinearizing yields:

$$0 = \int_I (x_{s,i,t} - p_{s,t})\, di - (1-\alpha_s)\, \pi_{s,t} \tag{31}$$

Given that optimizing firms are chosen at random at each time, and that $\tilde{k}$ is a gap relative to the sectoral capital:

$$\int_I \tilde{k}_{s,i,t} di = 0. \tag{32}$$

Hence, replacing (30) and (32) into (31) yields:

$$g_{s,t} = \frac{1-\alpha_s}{\alpha_s} \pi_{s,t}.$$

Calvo price setting implies that:

$$\begin{aligned} p_{s,j,t} &= \alpha_s x_{s,j,t} + (1-\alpha_s)\, p_{s,j,t-1} \\ &\Rightarrow E_t p_{s,j,t+1} = \alpha_s E_t x_{s,j,t+1} + (1-\alpha_s)\, p_{s,j,t} \end{aligned}$$

Rewriting:

$$\begin{aligned} E_t p_{s,j,t+1} &= \alpha_s E_t x_{s,j,t+1} + (1-\alpha_s)\, E_t x_{s,j,t} \\ E_t\left(p_{s,j,t+1} - p_{s,t+1}\right) &= \alpha_s E_t\left(x_{s,j,t+1} - p_{s,t+1}\right) + (1-\alpha_s)\, E_t\left(p_{s,j,t} - p_{s,t+1}\right) \\ E_t \tilde{p}_{s,j,t+1} &= (1-\alpha_s)\, E_t\left(\tilde{p}_{s,j,t} - \pi_{s,t+1}\right) + \alpha_s E_t \tilde{x}_{s,j,t+1} \end{aligned}$$

Using equation (30) and the fact that $g_{s,t} = \frac{1-\alpha_s}{\alpha_s}\pi_{s,t}$ to substitute in the above expression, we obtain:

$$\begin{aligned} E_t \tilde{p}_{s,j,t+1} &= (1-\alpha_s)\, E_t\left(\tilde{p}_{s,j,t} - \pi_{s,t+1}\right) + \alpha_s E_t \tilde{x}_{s,j,t+1} \\ &= (1-\alpha_s)\, E_t\left(\tilde{p}_{s,j,t} - \pi_{s,t+1}\right) + \alpha_s E_t\left[g_{s,t+1} - \psi_s \tilde{k}_{s,j,t+1}\right] \\ &= (1-\alpha_s)\, \tilde{p}_{s,j,t} - \alpha_s \psi_s \tilde{k}_{s,j,t+1} \end{aligned} \tag{33}$$

Extending Woodford's (2005) insight for multisector economies, the optimal quantity of investment in any period must depend only on j's relative capital stock, its relative price, and the economy's aggregate state. Thus, $\tilde{k}_{s,j,t+1}$ can be represented as a function of $\tilde{k}_{s,j,t}$, $\tilde{p}_{s,j,t}$. Hence, a firm j individual expectation (E_t^j) only involves periods at which the firm is not readjusting. We guess (and verify) that:

$$\tilde{k}_{s,j,t+1} = \kappa_{1,s}\tilde{k}_{s,j,t} - \kappa_{2,s}\tilde{p}_{s,j,t}, \tag{34}$$

where the parameters $\kappa_{1,s}, \kappa_{2,s}, \psi_s$ and function $g_{s,t}$ are to be determined.

The guess on capital implies that:

$$E_t\tilde{k}_{s,j,t+2} = \kappa_{1,s}\tilde{k}_{s,j,t+1} - \kappa_{2,s}E_t\tilde{p}_{s,j,t+1}.$$

Using equation (33), we get:

$$\begin{aligned} E_t\tilde{k}_{s,j,t+2} &= \kappa_{1,s}\tilde{k}_{s,j,t+1} - \kappa_{2,s}\left[E_t\tilde{p}_{s,j,t+1}\right] \\ &= \left(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s\right)\tilde{k}_{s,j,t+1} - \kappa_{2,s}\left(1-\alpha_s\right)\tilde{p}_{s,j,t} \end{aligned}$$

Using this last expression and equation (33) to substitute for (29) yields:

$$\begin{bmatrix} \beta\left(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s\right) - A \\ +\alpha_s\psi_s\left[1-(1-\delta)\beta\right]\frac{\theta(1+\gamma)}{\kappa\chi} \end{bmatrix}\tilde{k}_{s,j,t+1} = -\tilde{k}_{s,j,t} + \begin{bmatrix} \left[1-(1-\delta)\beta\right]\frac{\theta(1+\gamma)}{\chi\kappa}\left(1-\alpha_s\right) \\ +\kappa_{2,s}\left(1-\alpha_s\right)\beta \end{bmatrix}\tilde{p}_{s,j,t}.$$

Comparing with guess (34), we have:

$$\begin{bmatrix} \beta\left(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s\right) - A \\ +\alpha_s\psi_s\left[1-(1-\delta)\beta\right]\frac{\theta(1+\gamma)}{\kappa\chi} \end{bmatrix}\kappa_{1,s} = -1 \tag{35}$$

$$-\begin{bmatrix} \beta\left(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s\right) - A \\ +\alpha_s\psi_s\left[1-(1-\delta)\beta\right]\frac{\theta(1+\gamma)}{\kappa\chi} \end{bmatrix}\kappa_{2,s} = \begin{bmatrix} \left[1-(1-\delta)\beta\right]\frac{\theta(1+\gamma)}{\chi\kappa}\left(1-\alpha_s\right) \\ +\kappa_{2,s}\left(1-\alpha_s\right)\beta \end{bmatrix}. \tag{36}$$

Note that the system of dynamic equations (33) and (34) implies:

$$\begin{aligned} E_t\tilde{p}_{s,j,t+1} &= \left(1-\alpha_s\right)\tilde{p}_{s,j,t} - \alpha_s\psi_s\tilde{k}_{s,j,t+1} \\ &= \left(\left(1-\alpha_s\right) + \alpha_s\psi_s\kappa_{2,s}\right)\tilde{p}_{s,j,t} - \alpha_s\psi_s\kappa_{1,s}\tilde{k}_{s,j,t}, \end{aligned}$$

$$\begin{bmatrix} E_t\tilde{p}_{s,j,t+1} \\ \tilde{k}_{s,j,t+1} \end{bmatrix} = \begin{bmatrix} \left(1-\alpha_s\right) + \alpha_s\psi_s\kappa_{2,s} & -\alpha_s\psi_s\kappa_{1,s} \\ -\kappa_{2,s} & \kappa_{1,s} \end{bmatrix}\begin{bmatrix} E_t\tilde{p}_{s,j,t} \\ \tilde{k}_{s,j,t} \end{bmatrix}.$$

And we have both eigenvalues of the matrix inside the unit circle if and only if:

$$\begin{aligned} \kappa_{1,s} &< \left(1-\alpha_s\right)^{-1} \\ \kappa_{1,s} &< 1 - \kappa_{2,s}\psi_s \\ \kappa_{1,s} &> -1 - \frac{\alpha_s}{2-\alpha_s}\kappa_{2,s}\psi_s \end{aligned}$$

A.1.5 Optimal pricing rule

The optimal pricing equation (28) includes the term $E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \tilde{k}_{s,j,t+m}$, which varies according to each firm's capital accumulation history.

Using our solution guesses (30) and (34), we can rewrite this term as:

$$\begin{aligned} E_t^j \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \tilde{k}_{s,j,t+m} &= (1-(1-\alpha_s)\beta\kappa_{1,s})^{-1} \tilde{k}_{s,j,t} \\ &\quad -\kappa_{2,s} \frac{\beta(1-\alpha_s)}{(1-\beta(1-\alpha_s))(1-\beta(1-\alpha_s)\kappa_{1,s})} \tilde{p}_{s,j,t} \\ &\quad +\kappa_{2,s} \frac{(1-\alpha_s)\beta}{(1-\beta(1-\alpha_s))(1-\beta(1-\alpha_s)\kappa_{1,s})} \sum_{k=1}^{\infty} \beta^k (1-\alpha_s)^k E_t \pi_{s,t+k}. \end{aligned}$$

where we also used the fact that for any firm that doesn't readjust between times t and $t+m$:

$$\tilde{p}_{s,j,t+m} = \tilde{x}_{s,j,t} - \pi_{s,t+m} - ... - \pi_{s,t+1}$$

Hence, the optimal pricing equation (28) implies $\left(1+\frac{\theta(\gamma-(\chi-1))}{\chi}\right)\tilde{x}_{s,j,t} =$

$$\begin{aligned} &(1-\beta(1-\alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \begin{pmatrix} p_{t+m} + mc_{s,t+m} - p_{s,t+m} \\ +\left(1+\theta\frac{\gamma-(\chi-1)}{\chi}\right) \Sigma_{i=1}^m E_t \pi_{s,t+i} \end{pmatrix} \\ &-\frac{(1-\chi)(1+\gamma)(1-\beta(1-\alpha_s))}{\chi} \left[\begin{array}{c} (1-(1-\alpha_s)\beta\kappa_{1,s})^{-1} \tilde{k}_{s,j,t} \\ -\frac{\kappa_{2,s}\beta(1-\alpha_s)}{(1-\beta(1-\alpha_s))(1-\beta(1-\alpha_s)\kappa_{1,s})} \left[\begin{array}{c} \tilde{x}_{s,j,t} \\ -\sum_{k=1}^{\infty} \beta^k (1-\alpha_s)^k E_t \pi_{s,t+k} \end{array} \right] \end{array} \right]. \end{aligned}$$

Rearranging:

$$\begin{aligned} \phi_s \tilde{x}_{s,j,t} &= (1-\beta(1-\alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m (p_{t+m} + mc_{s,t+m} - p_{s,t+m}) \\ &\quad + \left(1+\theta\frac{\gamma-(\chi-1)}{\chi}\right)(1-\beta(1-\alpha_s)) \left\{ E_t \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \Sigma_{i=1}^m E_t \pi_{s,t+i} \right\} \\ &\quad - \frac{\kappa_{2,s}(1-\chi)(1+\gamma)\beta(1-\alpha_s)}{\chi(1-\beta(1-\alpha_s)\kappa_{1,s})} \sum_{k=1}^{\infty} \beta^k (1-\alpha_s)^k E_t \pi_{s,t+k} \\ &\quad - \frac{(1-\chi)(1+\gamma)}{\chi} \frac{(1-\beta(1-\alpha_s))}{(1-(1-\alpha_s)\beta\kappa_{1,s})} \tilde{k}_{s,j,t}, \end{aligned}$$

where $\phi_s = \left(1+\theta\frac{\gamma-(\chi-1)}{\chi} - \frac{\kappa_{2,s}(1-\chi)(1+\gamma)\beta(1-\alpha_s)}{\chi(1-\beta(1-\alpha_s)\kappa_{1,s})}\right)$.

Let's work through the expression in brackets $\{E_t \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m \Sigma_{i=1}^m E_t \pi_{s,t+i}\}$:

$$\begin{aligned}
m &= 0: \text{ does not contribute a term} \\
m &= 1: \beta(1-\alpha_s) E_t \pi_{s,t+1} \\
m &= 2: \beta^2 (1-\alpha_s)^2 (E_t \pi_{s,t+1} + E_t \pi_{s,t+2}) \\
&\quad ... \\
m &: \beta^m (1-\alpha_s)^m (E_t \pi_{s,t+1} + E_t \pi_{s,t+2} + ... + E_t \pi_{s,t+m})
\end{aligned}$$

Collecting coefficients on each inflation leads to:

$$\begin{aligned}
& \left[\beta(1-\alpha_s) + \beta^2 (1-\alpha_s)^2 + ... + \beta^m (1-\alpha_s)^m + ...\right] E_t \pi_{s,t+1} \\
& + \left[\beta^2 (1-\alpha_s)^2 + ... + \beta^m (1-\alpha_s)^m + ...\right] E_t \pi_{s,t+2} \\
& + ... \left[\beta^m (1-\alpha_s)^m + ...\right] E_t \pi_{s,t+m} + ... \\
= \; & \frac{\beta(1-\alpha_s)}{1-\beta(1-\alpha_s)} E_t \pi_{s,t+1} + \frac{\beta^2 (1-\alpha_s)^2}{1-\beta(1-\alpha_s)} E_t \pi_{s,t+2} + ... + \frac{\beta^m (1-\alpha_s)^m}{1-\beta(1-\alpha_s)} E_t \pi_{s,t+m} \\
= \; & \frac{1}{1-\beta(1-\alpha_s)} \sum_{m=1}^{\infty} \beta^m (1-\alpha_s)^m E_t \pi_{s,t+m} \\
& or \; \frac{1}{1-\beta(1-\alpha_s)} \sum_{k=1}^{\infty} \beta^k (1-\alpha_s)^k E_t \pi_{s,t+k}
\end{aligned}$$

Replacing the term above in term in brackets on the expression for prices, yields:

$$\begin{aligned}
\phi_s \tilde{x}_{s,j,t} &= (1-\beta(1-\alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m (p_{t+m} + mc_{s,t+m} - p_{s,t+m}) \\
&+ \left(1 + \theta \frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s} (1-\chi)(1+\gamma) \beta (1-\alpha_s)}{\chi (1-\beta(1-\alpha_s)\kappa_{1,s})}\right) \sum_{k=1}^{\infty} \beta^k (1-\alpha_s)^k E_t \pi_{s,t+k} \\
&- \frac{(1-\chi)(1+\gamma)}{\chi} \frac{(1-\beta(1-\alpha_s))}{(1-(1-\alpha_s)\beta\kappa_{1,s})} \tilde{k}_{s,j,t}
\end{aligned}$$

Recall that our guesses (30) and (34) take the form:

$$\tilde{x}_{s,j,t} = g_{s,t} - \psi_s \tilde{k}_{s,j,t}$$

$$\tilde{k}_{s,j,t+1} = \kappa_{1,s} \tilde{k}_{s,j,t} - \kappa_{2,s} \tilde{p}_{s,j,t}.$$

Hence, the solution for the pricing equation above implies that:

$$\begin{aligned}\phi_s g_{s,t} &= (1-\beta(1-\alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1-\alpha_s)^m (p_{t+m} + mc_{s,t+m} - p_{s,t+m}) \\ &\quad + \phi_s \sum_{k=1}^{\infty} \beta^k (1-\alpha_s)^k E_t \pi_{s,t+k},\end{aligned}$$

and ψ_s satisfies:

$$\phi_s \psi_s = \frac{(1-\chi)(1+\gamma)}{\chi} \frac{(1-\beta(1-\alpha_s))}{(1-(1-\alpha_s)\beta\kappa_{1,s})} \tag{37}$$

Note that this last equation can be solved for ψ_s as a function of $\kappa_{1,s}$ and $\kappa_{2,s}$.

Equation (37) along with equations (35) and (36) form a system of 3 equations and 3 unknowns, ψ_s, $\kappa_{1,s}$ and $\kappa_{2,s}$:

$$\begin{aligned}\phi_s \psi_s &= \frac{(1-\chi)(1+\gamma)}{\chi} \frac{(1-\beta(1-\alpha_s))}{(1-(1-\alpha_s)\beta\kappa_{1,s})} \\ [\beta(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s) - A + \alpha_s\psi_s\Xi]\kappa_{1,s} &= -1 \\ -[\beta(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s) - A + \alpha_s\psi_s\Xi]\kappa_{2,s} &= [\Xi(1-\alpha_s) + \kappa_{2,s}(1-\alpha_s)\beta]\end{aligned}$$

where:

$$\begin{aligned}\phi_s &= \left(1 + \theta\frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s}(1-\chi)(1+\gamma)\beta(1-\alpha_s)}{\chi(1-\beta(1-\alpha_s)\kappa_{1,s})}\right), \\ A &= \left[\beta + [1-(1-\delta)\beta]\left[\frac{\chi + (1+\gamma)(1-\chi)}{\kappa\chi}\right] - 1\right], \\ \Xi &= [1-(1-\delta)\beta]\frac{\theta(1+\gamma)}{\kappa\chi}.\end{aligned}$$

A.2 Firm-specific model – ϕ_s

The ϕ_s coefficients in equation (13) are given by:

$$\phi_s = \left(1 + \theta\frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s}(1-\chi)(1+\gamma)\beta(1-\alpha_s)}{\chi(1-\beta(1-\alpha_s)\kappa_{1,s})}\right),$$

where $\kappa_{1,s}$ and $\kappa_{2,s}$ are obtained from last section nonlinear system of 3 equations and 3 unknowns ψ_s, $\kappa_{1,s}$ and $\kappa_{2,s}$:

$$\begin{aligned}\frac{(1-\chi)(1+\gamma)}{\chi} \frac{(1-\beta(1-\alpha_s))}{(1-(1-\alpha_s)\beta\kappa_{1,s})} &= \phi_s \psi_s \\ -[\beta(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s) - A + \alpha_s\psi_s\Xi]\kappa_{1,s} &= 1 \\ -[\beta(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s) - A + \alpha_s\psi_s\Xi]\kappa_{2,s} &= [\Xi(1-\alpha_s) + \kappa_{2,s}(1-\alpha_s)\beta],\end{aligned}$$

where

$$
\begin{aligned}
A &= \left[\beta + [1-(1-\delta)\beta]\left[\frac{\chi + (1+\gamma)(1-\chi)}{\kappa\chi}\right] - 1\right] \\
\Xi &= [1-(1-\delta)\beta]\frac{\theta(1+\gamma)}{\kappa\chi}.
\end{aligned}
$$

Note that in a version of the model without capital accumulation, $\chi = 1$, ϕ_s simplifies to $\phi = (1+\theta\gamma)$ – which is familiar from new Keynesian DSGE models. Details of the derivation of these equations are available upon request.

A.3 The approximating 3-sector economy

Here we show that a model with three sectors, with suitably chosen degrees of price stickiness and sectoral weights, provides an extremely good approximation to the original 67-sector economy. We choose the sectoral weights and frequencies of price changes to match the following moments of the distribution of price stickiness from our baseline parametrization with 67 sectors: average frequency of price changes ($\overline{\alpha} = \sum_{s=1}^{S} f_s\alpha_s$), cross-sectional average of the expected durations of price spells ($\overline{d} \equiv \sum_{s=1}^{S} f_s\alpha_s^{-1}$), cross-sectional standard deviation of the expected durations of price spells ($\sigma_d = \sqrt{\sum_{s=1}^{S} f_s\left(\alpha_s^{-1} - \overline{d}\right)^2}$), skewness of the cross-sectional distribution of expected durations of price spells ($\mathcal{S}_d = \frac{1}{\sigma_d^3}\sum_{s=1}^{S} f_s\left(\alpha_s^{-1} - \overline{d}\right)^3$), and kurtosis of the cross-sectional distribution of expected durations of price spells ($\mathcal{K}_d = \frac{1}{\sigma_d^4}\sum_{s=1}^{S} f_s\left(\alpha_s^{-1} - \overline{d}\right)^4$).[33]

We present our findings in Figure 4. It shows the impulse response functions of real GDP to a nominal shock as described in Section 2.4, and in the approximating three-sector economy obtained with the moment-matching exercise described above.[34] The three-sector economy provides a very good approximation to all three multisector economies, which justifies our use of the approximating model to save on computational time.

[33] We have 5 degrees of freedom (2 weights and 3 frequencies of price change) to match 5 moments from the distribution of price stickiness in the 67-sector economy.

[34] The values of all other parameters are the same as in the baseline model.

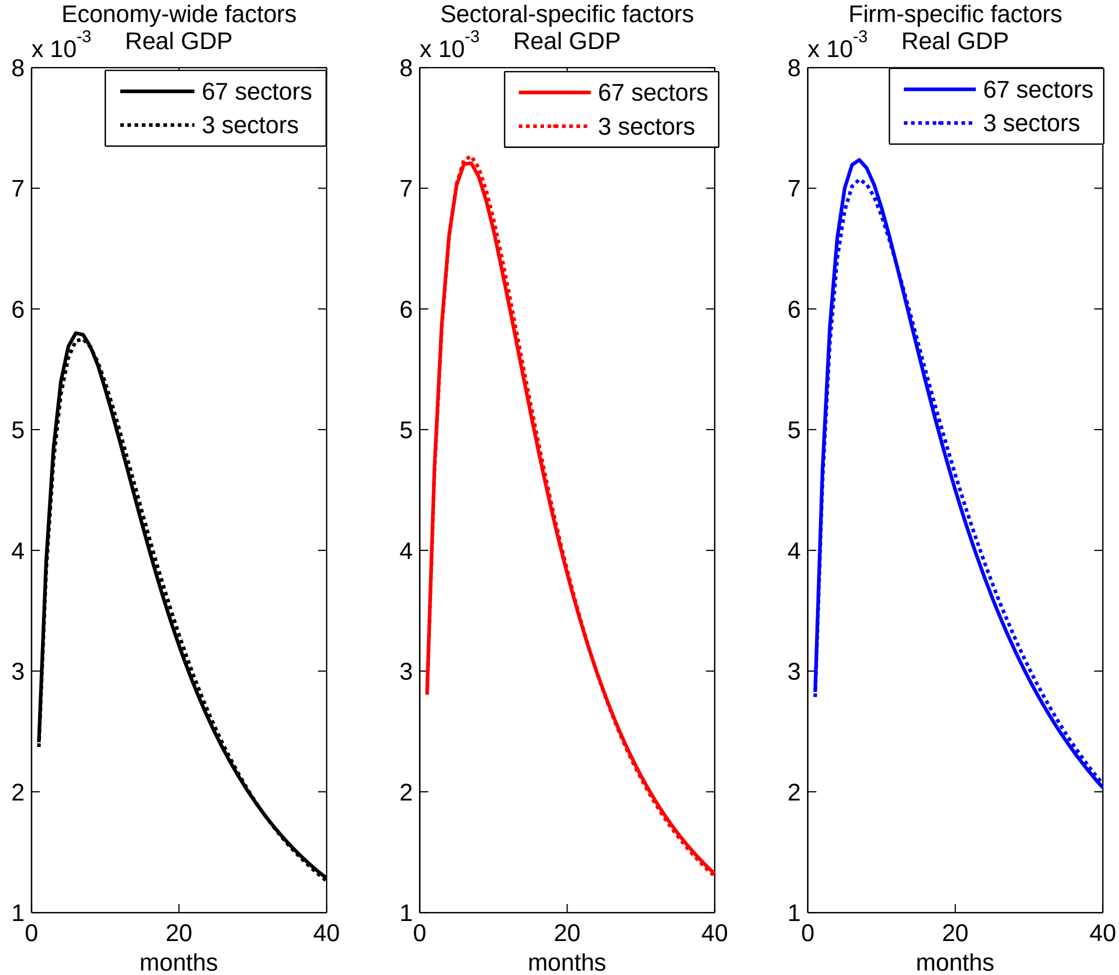

Figure 4: Impulse response functions of real GDP in 3- and 67-sector economies