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Marco Bonomo
Arnildo da Silva Correa
Marcelo Cunha Medeiros

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Marco Bonomo
Arnildo da Silva Correa
Marcelo Cunha Medeiros

Marco Bonomo
Escola de Pós-Graduação em Economia
Fundação Getúlio Vargas (EPGE/FGV)
Praia de Botafogo, nº 190
Botafogo
22253-900 - Rio de Janeiro, RJ – Brasil
marco.bonomo@fgv.br

Arnildo da Silva Correa
Banco Central do Brasil
Brasília, DF – Brasil
arnildo.correa@bcb.gov.br

Marcelo Cunha Medeiros
Departamento de Economia
Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio)
Rua Marquês de São Vicente, nº 225
Gávea
22451-900 - Rio de Janeiro, RJ – Brasil
mcm@econ.puc-rio.br

Estimating Strategic Complementarity in a State-Dependent Pricing Model*

Marco Bonomo
EPGE, Getulio Vargas Foundation

Arnildo da Silva Correa
Central Bank of Brazil

Marcelo Cunha Medeiros
Department of Economics, Pontifical Catholic University of Rio de Janeiro–Puc-Rio

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Abstract

The macroeconomic effects of shocks in models of nominal rigidity depend crucially on the degree of strategic complementarity among price setters. However, the empirical evidence on its magnitude is indirect and ambiguous: the one based on macroeconomic data suggest strong strategic complementarities in price-setting, which seems to be contradicted by some recent studies based on micro data. In this paper we estimate directly the degree of strategic complementarity based on individual price data underlying the CPI-FGV from Brazil for the 1996-2006 period, benefiting from large amount of macroeconomic variation in Brazilian sample during this period. Our identification strategy is to infer the degree of strategic complementarity from the relation between the frictionless optimal price and macroeconomic variables that results from a microfounded model. We assume that firms follow an asymmetric Ss pricing rule, which allows us to relate the price discrepancy (and the conditional probability of adjusment) to the change in the frictionless optimal price since the last adjustment date. As a consequence, assumptions for non-observable shocks lead to a relation between the probability of adjustments to conditional mean changes in the frictionless optimal price since the last adjustment. This relation allows us to directly estimate the degree of strategic complementarity from the occurrence of price adjustments. By explicitly assuming the Ss pricing rule, our methodology is able to disentangle the effect of strategic complementarity from the selection effect. The results, which are based on individual price changes and not on

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macro effects, indicate a substantial degree of strategic complementarity, contributing to reconcile micro and macro based evidence.

1 Introduction

In recent years there has been an increasing interest in the study of price rigidity. It became well understood that beyond the extent of price rigidity, the type of dynamic price setting policy matters for monetary policy effects. In particular, monetary policy shocks tend to have much less effect in models where price setters use state-dependent rules than in time-dependent models.[1] The reason is that in the former models firms that adjust prices tend to be further away from their profit maximizing price than firms that keep their price invariant—the so called selection effect—, leading to higher average price changes in response to shocks.[2] It has been known, since Ball and Romer (1990), that the effects of monetary policy are amplified when there are strategic complementarities in prices, that is, when individual desired prices increase with other prices for a given level of nominal aggregate demand. More recently, Gertler and Leahy (2008) have shown that strong enough strategic complementarity in prices may make aggregate price-level stickiness in a state-dependent model comparable to those in time-dependent models. Thus, the macroeconomic effects of shocks depend not only on the extent of price rigidity and on the details of the price-rigidity mechanism, but also on the structure of the underlying flexible price model.

More recently, the access to vast amounts of micro price data enabled the direct measurement of some of those micro features, such as the frequency and the size of price adjustments, while it has stimulated research strategies to unveil model features that are not directly observable. Price adjustments were found to be more frequent than previously thought (e.g. Bils and Klenow, 2004; Klenow and Krivtsov, 2008 and Nakamura and Steinson, 2008). Although it seems difficult to find a pricing model that fits perfectly the micro data, the evidence seems to be consistent with some state-dependency, as the frequency of price adjustments was shown to move with the macroeconomic environment (e.g. Dias, Marques, and Silva, 2007; Klenow and Krivtsov, 2008; Midrigan, 2011; Gagnon, 2009; Barros et al., 2010). Finally, some empirical work based on model simulation found indirect evidence of weak or no strategic complementarities in price-setting (Klenow and Willis, 2006; Burstein and Hellwig, 2007; Kryvtsov and Midrigan, 2012; and Bils, Klenow and Malin, 2009, 2012). As a result, this recent research seems to point out to a large gap between the features consistent with micro data—relatively high frequency of adjustments, state-dependency, and low degree of strategic

[1] The investigation of the macroeconomic consequences of state-dependent pricing rules goes back to Caplin and Spulber (1987) and Caballero and Engel (1991). More recently, optimal state-dependent pricing rules have been used to study macroeconomic effects (Almeida and Bonomo 2002, Golosov and Lucas 2007, Gertler and Leahy 2008, Midrigan 2009, Nakamura and Steinson 2009). Time-dependent models consider cases in which prices are fixed between adjustments, which have been used for much longer time (e.g. Taylor 1979 and Calvo 1983), as well as models where prices change continuously, as Mankiw and Reis (2004). Optimal time-dependent models appear in Bonomo and Carvalho (2004, 2010), and Reis (2006).

[2] Extreme contrasting cases are Caplin and Spulber (1987) model, for the state-dependent pricing rules and Calvo (1983) for time-dependent pricing rules. In the former case, the selection effect is so strong that it generates money neutrality. In Calvo (1983) there is no selection effect, since the firms that change their prices are randomly drawn.

complementarity—and those necessary to explain macroeconomic effects.

In this paper we revisit the issue of what degree of strategic complementarity is consistent with micro data. We use individual price data from Brazilian CPI of Getulio Vargas Foundation (CPI-FGV), following Gouvea (2007) and Barros et al (2010). We benefit from the large amount of macroeconomic variation in Brazilian sample. Studying how individual price variation depends on macroeconomic variables (e.g. Barros et al. 2010) is not the adequate way to address this matter, since price changes potentially reflect not only frictionless optimal price changes—that would enable us to assess the degree of price complementarities—, but also the frequency of price adjustments. We circumvent this problem by postulating that firms follow a two-sided Ss pricing rule. According to those pricing rules, adjustments are triggered whenever the price discrepancy—the difference between the price and its frictionless optimal level— attains a certain threshold. Thus, the pricing rule allows us to relate the price discrepancy (and the conditional probability of adjusment) to the change in the frictionless optimal price since the last adjustment date. As a consequence, assumptions for non-observable idiosyncractic and aggregate shocks imply a relation between probability of adjustments and the conditional mean of changes in the frictionless optimal price since the last adjustment. This relation allows us to directly estimate the degree of strategic complementarity using price adjustment observations.[3]

Our results indicate that the parameter measuring the sensitivity of relative price to real aggregate output[4] ranges from 0.03 to 0.10 for the economy as a whole, implying a substantial degree of strategic complementarity. In order to get a sense of the possible macro implications of this range of magnitude, notice that the values for the deep parameters assumed in Gertler and Leahy (2008) entail a degree of strategic complementarity of 0.08, inside the range of our estimations. In their state-dependent model, this magnitude is enough to ensure that monetary shocks have significant real effects.[5]

The degree of strategic price complementarity depends on several features of a model structure. Higher degree of strategic complementarity may result from the larger sensitivity of marginal cost to the firm's individual output and/or lower sensitivity to aggregate output. As argued by Woodford (2003), this is favored by short-run segmentation in labor and capital markets. Strategic complementarities in prices could also be increased by the dependence of output on intermediate goods, as in Basu (1995), or by relaxing the constant-elasticity between differentiated goods in favor of a formulation where the elasticity of demand decreases with the proportion of the product in the aggregate, as proposed by Kimball (1995).

In the current paper we are agnostic with respect to the relative importance of the different sources of strategic complementarity. Our methodology allows us to directly measure it, and does not rely on particular mechanisms. Thus, our work is complementary to those done through model simulation with the purpose of investigating whether some specific channel is consistent or not with some micro data set (e.g. Klenow and Willis 2006, Burstein and Hellwig 2007, and Krivtsov and Midrigan 2012). Klenow and Willis (2006)

[3]The resulting econometric model for price-adjustments is a non-standard ordered probit model, where the emerging autocorrelation and heteroskedasticity in the residuals are treated. The parameter measuring strategic complementarity is obtained from parameter restrictions in the structural model.

[4]It corresponds to one minus the sensitivity of individual price to the average price.

[5]Since we do not have a complete model, we refrain from evaluating numerically the aggregate implications of our estimation.

investigate whether price strategic complementarity engendered by variable desired markups due to Kimball's preferences are consistent with CPI micro data on prices. They pointed out that reconciling a large degree of strategic complementarity with the large variability of relative prices would require an implausibly large variance of idiosyncratic shocks. Burstein and Hellwig (2007) conclude that price and market share data from a large chain of supermarkets is consistent with a moderate degree of price complementarity.[6,7] Krivtsov and Midrigan (2012) found out that low variability of inventories at business cycle frequencies are not easily reconcilable with marginal cost rigidity.

Bils, Klenow and Malin (2009, 2012) approach is more related to ours, since they also proposed a methodology to disentangle the frequency of price adjustments from variations in the frictionless optimal price. Their method consists in constructing a new statistics—reset price inflation—, that would help to evaluate competing models of price rigidity. Their theoretical reset price measure corresponds to the average price that each firm would like to have in case of adjustment. It turns out that if rules are two sided Ss, as we assume, their theoretical reset price is identical to the frictionless optimal price (plus a constant). However, their approach does not allow them to recover that measure. They use BLS CPI micro data to construct, instead, a related auxiliary variable—empirical reset price inflation[8]—, which differs from the theoretical counterpart when the selection effect is present. They conclude that adding strategic complementarities to a model does not help it to match the empirical reset price inflation statistics.[9]

Our results differ markedly from those obtained by them. By explicitly assuming Ss pricing rules, our methodology disentangles the effect of strategic complementarity from selection. Another distinctive feature of our approach is that it is based only on the supply side of the economy, and does not depend on consumer Euler equation and monetary policy rule specifications. So, it should provide results that are not affected by possible mispecifications of the demand side.

In order to investigate whether the difference in methodology or in the data is driving the difference in results, we replicate their methodology with our data. We found a similar pattern of empirical reset inflation with the Brazilian data. Therefore, if we were to use their methodology with our data, we would arrive to the same conclusion as them. However, our frictionless price estimation allow us to recover frictionless price inflation, which corresponds to theoretical reset price inflation, and it is substantially more persistent than the empirical reset price inflation. The impulse response function (IRF) of our frictionless price is similar to their IRF for theoretical reset price simulated with base on a Ss model with strategic complementarities. We conclude that the difference must come from the methodology. As suggested by their own empirical results, the specification of the demand side can make a big difference.[10] By contrast, our methodology focuses on

[6] They assess that the value of the parameter measuring the sensitivity of an individual price to the average price is about 0.6.

[7] Their simulation exercises also rely on targeting the fraction of menu costs as a proportion of firm revenues in 1%.

[8] The empirical reset price is defined to be the new price set by adjusters, and it is imputed for non-adjusters by adjusting their last period reset price by the inflation in adjusters' reset price.

[9] They explicitly examine whether strategic complementarities are consistent with empirical reset price inflation statistics in the 2009 NBER working paper version.

[10] When they allowed money supply to respond to real aggregate demand, the state-dependent model with strategic com-

estimating the frictionless optimal price equation, which does not depend on the demand side specification.

The remaining parts of the paper is organized as follows. In the next section we derive the equation for the frictionless optimal price from a structural model. The frictionless optimal price is determined by macroeconomic variables, aggregate and idiosyncratic shocks. In this particular model, we are able to relate the sensitivity of the frictionless optimal price with respect to the aggregate price ζ to preference and technology parameters. We then notice that although different settings would imply different expressions for ζ, our main interest is to estimate ζ. Since we observe actual prices and not the frictionless optimal price, in order to relate them, we postulate that firms follow a two-sided Ss adjustment rule.

In Section 3 we specify processes for idiosyncratic and aggregate shocks in order to relate the likelihood of adjusments to changes in the macroeconomic variables and accumulated aggregate shocks since the last adjustment time. The resulting ordered probit model is estimated by quasi-maximum likelihood. We use the panel structure of our estimation to identify the common aggregate shocks to the economy and to estimate the model parameters. Then, we are able to recover the parameters of interest, in particular ζ and the size of upward and downward adjustments, from the estimated parameters.

Section 4 describes the individual price adjustment data set underlying the Brazilian CPI from Getulio Vargas Foundation (CPI-FGV) and the treatments we apply to it. In additional to conventional treatments, we follow Eichenbaum et al. (2012) by excluding problematic price quotes from our sample.

Results are presented in section 5. Prices are found to be highly complementary in our baseline estimation. This result survive the several robustness checks we perform. We also evaluate our results with base on statistics for the frictionless inflation, the empirical reset price inflation and the actual inflation process, and compare to those obtained by Bils, Klenow and Malin (2009).

The last section concludes.

2 A model of state-dependent pricing

In this section we first derive the frictionless optimal price equation to be estimated from a standard structural model with price flexibility, and then we superimpose price rigidity through a symmetric Ss rule, which will be essential to link the frictionless optimal price with the actual price data.

2.1 Frictionless optimal price[11]

The structural model is similar to Woodford (2003). Households obtain utility from consumption goods and disutility from supplying labor; firms in a monopolistically competitive environment produce differentiated goods from two types of inputs: labor and a foreign intermediate good. In the (segmented) labor market, households and firms behave competitively.

plementarity turned out to fit better the impulse response function of reset price inflation than the model without strategic complementarity. However, they reject the model with endogenous money because it generates a too smooth inflation process.

[11] See Appendix A for a detailed derivation of the frictionless optimal price from fundamentals.

The representative household has intertemporal utility given by:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U_t \frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \int_0^1 \frac{V_{i,t} L_{i,t}^{1+\delta}}{1+\delta} di \right] \right\},$$

where C is the consumption of a composite good, L_i is the quantity of labor of type i supplied, U is an aggregate demand shock and V_i represents a shock in the disutility of type of work i. The composite good over which utility is defined is given by

$$C_t = \left[\int_0^1 C_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/\theta-1}. \tag{1}$$

where C_i is the consumption of variety i and $\theta > 1$ is the elasticity of substitution among varieties.

In this setting, the demand for an individual product has the familiar form:

$$C_{i,t} = C_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}, \tag{2}$$

where P_i is the price of variety i and the price index P is given by:

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}. \tag{3}$$

The optimal quantity of type i of labor supplied satisfies:

$$\frac{V_{i,t} L_{i,t}^{\delta}}{U_t C_t^{-\frac{1}{\sigma}}} = \frac{W_{i,t}}{P_t} \tag{4}$$

where W_i is the wage of type i of labor.

There is a continuum of monopolistically competitive firms, each one supplying goods from one variety. We assume that each firm has the Cobb-Douglas production function on a specific labor and a common foreign input:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} M_t^{(1-\alpha)}, \tag{5}$$

where Y_i is the firm's product, A_i is a productivity factor and M is a foreign input used in the production process. We associate the nominal exchange rate $\tilde{E}_t$ with the price of the common foreign input, and assume it to be exogenous.

If price adjustment frictions were absent, a producer i would choose a profit maximizing price $P_{i,t}^*$ that satisfies the usual markup rule:

$$\frac{P_{i,t}^*}{P_t} = \mu \psi(Y_{i,t}, Y_t, E_t; U_t, V_{i,t}, A_{i,t}), \tag{6}$$

where $\psi(.)$ is the real marginal cost, Y_t is the aggregate output, E_t is the real exchange rate and $\mu \equiv \theta/(\theta-1)$ is the firm's desired markup.

It is shown in the appendix that the real marginal cost function is given by:

$$\psi(Y_{i,t}, Y_t, E_t; \nu_{i,t}, \upsilon_t, A_{i,t}) = \overline{\kappa} A_{i,t}^{-\frac{1+\delta}{1+\delta(1-\alpha)}} Y_{it}^{\frac{\alpha\delta}{1+\delta(1-\alpha)}} Y_t^{\frac{\alpha\sigma^{-1}}{(1+\delta(1-\alpha))}} E_t^{\frac{(1-\alpha)+\delta(1-\alpha)}{1+\delta(1-\alpha)}} V_{i,t}^{\frac{\alpha}{1+\delta(1-\alpha)}} U_t^{-\frac{\alpha}{1+\delta(1-\alpha)}} \qquad (7)$$

where

$$\overline{\kappa} \equiv \lambda \left[\frac{(1-\alpha)}{\alpha}\right]^{\frac{\alpha^2\delta(1-\alpha)}{1+\delta(1-\alpha)}}$$

Substituting the marginal cost expression (7) into the markup rule (6) and taking the log of the resulting expression yields the frictionless optimal price equation that we want to estimate:

$$p_{i,t}^* = \kappa + \zeta \mathcal{Y}_t + (1-\zeta)\, p_t + \phi e_t + \tilde{\xi}_t + \tilde{a}_{i,t} \qquad (8)$$

where lowercase variables represent log of the corresponding uppercase variables, $\mathcal{Y}_t$ is the (log of) nominal aggreate demand, and shocks and parameters are combinations of the structural parameters and shocks:

$$\begin{aligned}
\zeta &\equiv \frac{\alpha\left(\delta+\sigma^{-1}\right)}{1+\delta\left[(1-\alpha)+\alpha\theta\right]} \\
\phi &\equiv \frac{(1+\delta)(1-\alpha)}{1+\delta(1-\alpha)+\theta\delta\alpha} \\
\tilde{a}_{i,t} &\equiv -\frac{1+\delta}{1+\delta(1-\alpha)+\theta\delta\alpha} a_{i,t} + \frac{\alpha}{1+\delta(1-\alpha)+\theta\delta\alpha} \upsilon_{i,t} \\
\tilde{\xi}_t &\equiv -\frac{\alpha}{1+\delta(1-\alpha)+\theta\delta\alpha} u_t \\
\kappa &\equiv \frac{1+\delta(1-\alpha)}{1+\delta(1-\alpha)+\theta\delta\alpha} \log \overline{\kappa}.
\end{aligned}$$

Observe that when $\zeta < 1$, the optimal price the firm would like to charge depends positively on the prices set by its competitors (in our model represented by p_t), a condition we refer to as strategic complementarities in price setting. The degree of strategic complementarity depends positively on σ (elasticity of intertemporal substitution) and negatively on α (elasticity of product with respect to labor), and θ (elasticity of substitution among alternative varieties). Assuming $\sigma^{-1} \geq 1$, as usual in the literature, is sufficente for a positive relation between the degree of price complementarity and δ (the inverse of the Frish elasticity of labor supply)[12].

Notice that the frictionless optimal price equation (8) is valid for a set of different structural models, that includes the one we just laid out. Different models in this set imply different relations between the parameters of the frictionless optimal price equation (8) and the deep structural parameters. For example,

[12] The exact necessary condition on σ is: $\sigma < 1 - \alpha + \theta\alpha$.

if we had assumed homogenous labor markets, our parameter of interest ζ would be given by

$$\zeta = \frac{\alpha \left(\delta + \sigma^{-1}\right)}{1 + \delta \left(1 - \alpha\right)}$$

which is larger than the one obtained under segmented labor market hypotheses if the deep structural parameter values are kept the same. Although those alternative assumptions are important if one wants to assess the degree of price complementarities with base on reasonable calibrated values for the structural parameters, they will not influence our estimation of (8), and in particular our estimation of ζ.

2.2 The pricing rule

We would like to estimate equation (8), but we do not observe the frictionless optimal price $p^*_{i,t}$. We only observe the actual price $p_{i,t}$. To bridge this gap we need a pricing rule. We assume that firms follow a Ss pricing policy parameterized by (s, c, S). The state variable is the discrepancy between the actual and the frictionless optimal price:

$$r^*_{i,t} \equiv p_{i,t} - p^*_{i,t}.$$

The parametes s and S are the lower and the upper bounds for $r^*_{i,t}$, and c is the target point. Although we do not derive the pricing rule here, it can be rationalized by the existence of menu costs for price adjustment and additional conditions for the $p^*_{i,t}$ process (e.g. a brownian motion)[13]. For our purposes, knowing the format of the pricing policy suffices to derive the econometric model to be estimated.

According to this pricing rule, while price discrepancy $r^*_{i,t}$ is inside the range (s, S), the firm maintains its price fixed. When $r^*_{i,t}$ reaches the threshold s, however, $p^*_{i,t}$ is sufficiently above the actual price charged and the firm increase its price. Similarly, when the upper threshold S is reached, the firm decreases its price. In case of price changes, the price discrepancy variable $r^*_{i,t}$ is set equal to c.[14]

Formally, an individual price will be adjusted by

$$\begin{cases} c - s & \text{if} \quad r^*_{it} \leq s \\ 0 & \text{if} \quad s \leq r^*_{it} \leq S \\ -\left(S - c\right) & \text{if} \quad r^*_{it} \geq S \end{cases} \tag{9}$$

[13] In principle we could include the conditions that determine the optimal rule in the estimation, and estimate the menu cost instead of the pricing rule. However since the $p^*_{i,t}$ for one firm depends on other firms' prices, this would involve solving a cumbersome fixed point problem in the estimation process.

[14] Notice that the value of c, is in general different from zero in optimized policies. As the firm knows that it will maintain the actual price fixed for some time interval, it may be optimal to set $p_{i,t}$ somewhat above $p^*_{i,t}$, if inflation is positive.

3 The empirical model

In this section we develop the empirical model for estimating the frictionless optimal price equation (8). For that purpose, we use the frictionless optimal price equation (8) and the pricing rule (9), and assume a certain structure for the idiosyncratic and aggregate shocks. The resulting model is a non-standard ordered probit that we estimate by quasi-maximum likelihood method.

First notice that $p^*_{i,t}$ is non-observable and depends on the aggregate observable variables p_t, $\mathcal{Y}_t$ and e_t, and on the unobservable aggregate and idiosyncratic shocks $\tilde{\xi}_t$ and $\tilde{a}_{i,t}$. For convenience, we will rewrite (8) as

$$p^*_{i,t} = \kappa + \boldsymbol{x}_t'\boldsymbol{\beta} + \tilde{\xi}_t + \tilde{a}_{i,t}, \tag{10}$$

where $\boldsymbol{x}_t = (\mathcal{Y}_t, p_t, e_t)'$ represents the vector of observable aggregate variables and $\boldsymbol{\beta} = [\zeta, (1-\zeta), \phi]'$ is the vector containing their coefficients.

Since price adjustments are triggered by the level of the price discrepancy variable $r^*_{i,t}$, we would like to write it in terms of the changes in the observable variables and in the shocks since the last adjustment date. In order to do that, first we write:

$$\begin{aligned} r^*_{i,t} &\equiv p_{i,t} - p^*_{i,t} \\ &= p_{i,\tau_{i,t}} - p^*_{i,t} \\ &= c + p^*_{i,\tau_{i,t}} - p^*_{i,t}, \end{aligned} \tag{11}$$

where $\tau_{i,t}$ is firm's i last adjustment date prior to t. The equality in the second line follows from the fact that, with price rigidity, the current price is the one of the last adjustment date. The one in the third line results from our Ss pricing rule assumption: under this type of rule, adjustments are always made to a constant proportion of the frictionless optimal price. This last property of Ss rules is key to our identification strategy, since it allows us to relate the price discrepancy to the change in the frictionless optimal price.

Replacing the $p^*_{i,t}$ specification (10) into the price discrepancy equation (11) yields:

$$\begin{aligned} r^*_{i,t} &= c + \left(\kappa + \boldsymbol{x}'_{\tau_{i,t}}\boldsymbol{\beta} + \tilde{\xi}_{\tau_{i,t}} + \tilde{a}_{i,\tau_{i,t}}\right) - \left(\kappa + \boldsymbol{x}_t'\boldsymbol{\beta} + \tilde{\xi}_t + \tilde{a}_{i,t}\right) \\ &= c + \left(\boldsymbol{x}_{\tau_{i,t}} - \boldsymbol{x}_t\right)'\boldsymbol{\beta} + \left(\tilde{\xi}_{\tau_{i,t}} - \tilde{\xi}_t\right) + \left(\tilde{a}_{i,\tau_{i,t}} - \tilde{a}_{i,t}\right) \\ &= c - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \left(\tilde{\xi}_t - \tilde{\xi}_{\tau_{i,t}}\right) - \left(\tilde{a}_{i,t} - \tilde{a}_{i,\tau_{i,t}}\right) \end{aligned} \tag{12}$$

where $\boldsymbol{z}_{i,t} \equiv \boldsymbol{x}_t - \boldsymbol{x}_{\tau_{i,t}}$. Notice that although only macroeconomic variables are contained in $\boldsymbol{z}_{i,t}$, this vector of variables is not the same for all firms at time t, because $\tau_{i,t}$, the last adjustment date prior to t, varies across firms. Since firms change their prices at different points in time, this cumulative variation differs across firms for each time. It is precisely this variation in price spells that enables us to estimate the model with only aggregate variables as regressors.

The model estimation requires specifying the aggregate and idiosyncratic shock processes. In order to make it tractable, we assume that the aggregate shock $\tilde{\xi}_t$ follows a random walk:

$$\tilde{\xi}_t = \tilde{\xi}_{t-1} + v_t, \quad v_t \sim \mathsf{IID}(0, \sigma_v^2). \tag{13}$$

This assumption could be motivated by the highly persistent common component of inflation found in the recent literature (e.g. Boivin, Giannoni and Mihov 2009).

In addition, we assume that the idiosyncratic shock $\tilde{a}_{i,t}$ is given by

$$\tilde{a}_{i,t} = a_i + a_{i,t},$$

where a_i is an individual fixed effect component and[15]

$$a_{i,t} = \eta + a_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2). \tag{14}$$

Therefore, our model also takes into account that there may be individual fixed specificities in the price of each item, but since the idiosyncratic component appears in difference, these individual effects will not matter for the estimation.

Substituting the specification of shocks into equation (12), we obtain[16]:

$$\begin{aligned} r_{i,t}^* &= c - \eta\delta_{i,t} - \boldsymbol{z}_{i,t}'\boldsymbol{\beta} - \sum_{j=t-\delta_{i,t}+1}^{t} v_j - u_{i,t} \\ &= c - \eta\delta_{i,t} - \boldsymbol{z}_{i,t}'\boldsymbol{\beta} - \sum_{j=1}^{T} \gamma_j d_{i,t}(j) - u_{i,t}, \end{aligned}$$

where

$$\begin{aligned} \delta_{i,t} &\equiv (t - \tau_{i,t}), \\ d_{i,t}(j) &= \begin{cases} 1, & \text{if } j \in [t - \delta_{i,t} + 1, t] \\ 0, & \text{otherwise} \end{cases}. \end{aligned} \tag{15}$$

and

$$u_{i,t} \equiv \sum_{\boldsymbol{j=t-\delta_{i,t}+1}}^{\boldsymbol{t}} \varepsilon_{i,j} \sim \mathbb{N}(0, \delta_{i,t}\sigma^2).$$

[15] In the main specification we assume that $a_{i,t}$ is a random walk with drift because we would like to take into account that productivity grows over time. But, given the recent evidence that idiosyncratic shock is short-lived, we also estimate the model assuming that $a_{i,t}$ is a white-noise, i.e., $a_{i,t} = \varepsilon_{i,t}$ and $\varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2)$. See subsection 5.3.

[16] See in Appendix B the derivation of this equation.

Notice that we treat aggregate and idiosyncratic shocks differently. We will be able to identify the aggregate shocks $\{v_t\}_{t=1}^{T}$ through the cross-section of prices. Therefore, we define a dummy variable specific to each firm at each time which lights up during the period that starts in the previous adjustment and finishes in the current period t. Note that the coefficient γ_j does not depend on i, since the effect of the aggregate shock will be the same for all firms that suffer its impact. Those are the firms that had their last adjustment before the shock took place, i.e., firms that have not incorporated the shock in their current prices. By controlling for these common shocks in our model (in addition to the other observable regressors), we want to make sure that two firms that change prices at the same time reprice together not because they were affected by the same unobserved aggregate shock, but because of some kind of strategic interaction between them.

For the idiosyncractic shock, what matters is its distribution. We sum the idiosyncratic shocks $\{\varepsilon_{i,j}\}$ into the new shock $u_{i,t}$, which inherits its distribution properties from $\{\varepsilon_{i,j}\}$. Also observe that $\{u_{i,t}\}$ are naturally autocorrelated. However, given our assumptions, we know the autocorrelation structure—$u_{i,t}$ is a moving average $MA(\delta_{i,t}+1)$ process—and we use this information in the estimation procedure.

Our estimation is based on the observation of price adjustment ocurrences and their signal. We create the auxiliary variable $r_{i,t}$ which assumes values 0, 1 and -1 if no-adjustment, a price increase or a price decrease is observed, respectively:

$$r_{i,t} = \begin{cases} 1, & \text{if } \; p_{i,t} > p_{i,t-1} \\ 0, & \text{if } \; p_{i,t} = p_{i,t-1} \\ -1, & \text{if } \; p_{i,t} < p_{i,t-1} \end{cases} . \tag{16}$$

Defining $\boldsymbol{w}_{i,t} = (\delta_{i,t}, \boldsymbol{z}'_{i,t}, \boldsymbol{d}'_{i,t})'$, where $\boldsymbol{d}_{i,t} = (d_{i,t}(1), ..., d_{i,t}(T))'$, and using the pricing rule, we can obtain the probability of observing a price increase from the cumulative distribution of the idiosyncractic shocks since the last adjustment date. Straightforward algebra yields:

$$\begin{aligned} Pr[r_{i,t}=1|\boldsymbol{w}_{i,t}] &= Pr[r^*_{i,t} \leq s|\boldsymbol{w}_{i,t}] \\ &= Pr[c - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T}\gamma_j d_{i,t}(j) - u_{i,t} \leq s|\boldsymbol{w}_{i,t}] \\ &= Pr\left[\frac{u_{i,t}}{\sqrt{\delta_{i,t}}\sigma} \geq \frac{c - s - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T}\gamma_j d_{i,t}(j)}{\sqrt{\delta_{i,t}}\sigma}\right] \\ &= 1 - \Phi\left(\frac{c-s}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{i,t}(j)}{\sqrt{\delta_{i,t}}}\right) \\ &= 1 - \Phi\left(\pi_1 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j \ddot{d}_{i,t}(j)\right) \end{aligned} \tag{17}$$

where variables with two dots represent the variables divided by $\sqrt{\delta_{i,t}}$, the parameters with tilde means that the parameters are scaled by σ, $\Phi(.)$ is the cumulative distribution function of a standard normal variable and $\pi_1 = (c - s)/\sigma$. In the third line we have used the fact that $u_{i,t}$ is independent of $\boldsymbol{w}_{i,t}$.

Likewise, we can derive the probability of observing the other two possible outcomes for $r_{i,t}$. This results in the following ordered probit model for price changes:

$$Pr[r_{i,t} = 1|\boldsymbol{w}_{i,t}] = 1 - \Phi\left(\pi_1 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T} \tilde{\gamma}_j \ddot{d}_{i,t}(j)\right)$$

$$Pr[r_{i,t} = 0|\boldsymbol{w}_{i,t}] = \Phi\left(\pi_1 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T} \tilde{\gamma}_j \ddot{d}_{j,t}\right) - \Phi\left(\pi_0 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T} \tilde{\gamma}_j \ddot{d}_{i,t}(j)\right) \quad (18)$$

$$Pr[r_{i,t} = -1|\boldsymbol{w}_{i,t}] = \Phi\left(\pi_0 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T} \tilde{\gamma}_j \ddot{d}_{i,t}(j)\right)$$

for $t = 1, ..., T$ and $i = 1, ..., N$, where $\pi_0 = \frac{c-S}{\sigma}$.

The probit model is estimated by quasi-maximum likelihood method, which guarantees a consistent estimation of the parameters under fixed T and $N \to \infty$. Inference is carried out using a robust variance-covariance matrix for autocorrelation and heteroskedasticity. Details are given in Appendix B.

3.1 Identification of strategic complementarity and pricing rule parameters

The strategy to recover the structural parameter ζ measuring strategic complementarity in the model arises naturally from the restrictions on the frictionless optimal price parameters and the ordered probit model.

Notice that from equation (8) and the probit model, we have:

$$\tilde{\beta}_1 = \frac{\zeta}{\sigma} \quad (19)$$

and

$$\tilde{\beta}_2 = \frac{1 - \zeta}{\sigma}. \quad (20)$$

Then, by dividing equation (19) by (20) and isolating ζ we obtain:

$$\zeta = \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}. \quad (21)$$

Similarly, we can estimate the variance of shocks hitting the firms. To obtain σ, substitute equation (21) into (19):

$$\sigma = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}. \tag{22}$$

Since we are able to write ζ and σ as a function of the probit model's parameters, we can use the Delta method to construct confidence intervals for them. Details are given in Appendix B.

Parameters π_0 and π_1 of the probit model are combinations of the pricing rule's parameters and the variance of shocks. Even though we cannot obtain all parameters of the pricing rule in isolation, once we estimate the variance of shocks we can estimate the adjustment sizes and the inaction band size. They are obtained respectively as

$$c - s = \pi_1 \sigma, \qquad c - S = \pi_0 \sigma \quad \text{and} \quad S - s = (\pi_1 - \pi_0)\,\sigma. \tag{23}$$

4 Data

In this section we describe the data set that we use in our estimations and calculate some statistics of price changes, such as frequency, average price change etc. These statistics will be useful as a guideline to our methodology. For instance, we compare some of these statistics in the data to those estimated by the probit model, and this exercise can provide some sense of goodness of fit of our model. We also relate some of them to the estimated princing rule.

4.1 Data set description

The data set used here consists of primary information of price quotes of individual products collected and used by the Brazilian Institute of Economics of Getulio Vargas Foundation (FGV-IBRE) to compute the consumer price index (CPI-FGV). The CPI-FGV has wide coverage and it is the oldest Brazilian consumer price index, being calculated since 1944.

The data set contains information on prices at the firm level collected in the 12 largest Brazilian metropolitan regions, even though the coverage has changed during our sample period.[17] Currently, the CPI-FGV comprises 456 products and services grouped into seven different sectors: Apparel; Education and Recreation; Food; Housing; Medical and Personal Care; Transportation; and Other Goods and Services. The weight of each product or service used in computing the index is based in a survey of household expenditures.[18]

For some products the data are sampled every ten days, while for the remaining ones prices are collected monthly. We refer to the most disaggregated level of data as an *item*. Each item is identified by a set of

[17] Up to the end of 2000, the coverage comprised only the metropolitan regions of Rio de Janeiro and São Paulo. After January 2001, ten other cities were included in the survey: Belo Horizonte, Brasília, Porto Alegre, Recife, Salvador, Belém, Curitiba, Florianópolis, Fortaleza and Goiania. At the beginning of 2005 the survey was suspended in the last five cities.

[18] The Survey of Family Budget (POF) is conducted by the Brazilian Institute of Geography and Statistics (IBGE), covering families with income up to 33 times the minimum wage.

very specific characteristics, such as brand, model, packaging, type/variety that is sold in a particular outlet, in a specific city.[19] The number of items collected each month is not constant. There is variation due to item exclusions and inclusions over time. However, there is never substitution of an item by a similar one. Therefore, it is possible to follow the same item over time. When the price of a specific item is no longer collected, its price is tagged as missing. Thus, a recorded price change—excluded sampling errors common to some products that we discuss below—should reflect an actual price change in the same item.

4.2 Data sample and some statistics

We have a very representative sample of the overall CPI-FGV (around 85%), containing 243 categories of products and services for a period of approximately 11 years, from 1996 to 2006. All the seven sectors are very well represented.

We start from the original full sample with approximately 7.4 million price quotes and 122 thousand quote lines. The original sample was treated in order to have a set of data more suited to our goals.

Recently Eichenbaum et al. (2012) raised the possibility that most of papers using price microdata potentially suffer from measurement errors due to problematic price quotes. In particular, they argue that the evidence suporting the view that firms make many small price changes is fragile, plagued by a variety of measurement errors. Aiming to avoid this type of problem, the first treatment we apply to our original sample is the exclusion of products identified with potential problematic price quotes.[20] Detailed analysis of the procedures adopted in the collection of prices in our database identified five broad categories of potential sources of measurement errors. Products identified as falling into any of these categories were excluded from our sample. Appendix D documents the potential sources of measurement errors and lists the products excluded. After those exclusions we were left with 178 products and services with all seven sectors still represented, accounting for 40.4% of all prices in the CPI-FGV. For robustness, in subsection 5.3 we also estimate our model without those exclusions.

Second, we would like the same sample interval for all items. Thus, in order to have monthly sample intervals for all items in our sample, we keep only the first price quote of the month whenever a item has price quotes collected every 10 days. Third, items with very short price trajectories—less than 18 observations or with more of 30% missing observations—were excluded. We further refined our sample by treating the remaining gaps. The gaps with only one missing observation were filled using the price collected in the immediately preceding month. Also, gaps with up to three missing observations, when preceded and succeeded by the same price, were filled with the last price available. In case of four or more missing observations, we maintained the longest uninterrupted part of the price trajectory.

Fourth, retail prices are characterized by a significant number of temporary price decreases (sales or promotions). Following most of the related literature (Golosov and Lucas, 2007; Midrigan, 2011 and others),

[19] For example, type I black beans of the Combrasil brand, sold in a 1kg package in the outlet number 16,352, in Belém.

[20] These potential measurement errors do not affect the aggregate price index, but are important for micro studies based on items price paths.

we treated observations identified as sales prices. Because sales are not explicitly classified in our database, we used the following algorithm to identify them. If a price decrease was large enough and reversed in the following months to levels near the one prevailing before the change, we classified that observation as a price sale.[21] Following Midrigan (2011), to deal with large price changes in a V shape, we repeated the algorithm three times. When a "sale" was identified, the price was replaced by the price collected in the immediately preceding period. For robustness, we also estimate our model without treating sales (see subsection 5.3). [22]

We also treat outliers, discarding an observation when it was 10 times larger or 10 times smaller than the previous observation. Finally, to avoid left-censored spells, for all price trajectories we dropped the observations before the first observed price change.

After this treatment we were are left with a final data sample with approximately 2.2 million observations and 46.2 thousand price trajectories into the seven sectors of the CPI-FGV.

Table 1: Some statistics of price changes

Statistics	Estimated value
Average frequency of price change	0.49
Average absolute size of price changes (%)	15.3
Average size of price increase (%)	15.3
Average size of price decrease (%)	16.0
Percentage of price decreases	44.1

Notes: 1) p is defined as the natural logarithm of the item price.
2) All statistics are calculated based on unweighted price changes.

Table 1 presents some statistics related to price adjustments in our final sample. All statistics are calculated using unweighted price changes. Price adjustments are much more frequent than in the US (see also Gouvea, 2007 and Barros et al., 2010 for detailed analysis of Brazilian price-setting statistics). Similar to the evidence from the US CPI micro data (Nakamura and Steinsson, 2008; Klenow and Krivtsov, 2008 and Klenow and Malin, 2010), we also find that price changes in the Brazilian sample are large on average (approximately 15%), with slightly larger price decreases (16%) than price increases (15.3%).

5 Estimation and results

We use the data set of individual prices described in the previous section, combined with aggregate data, to estimate the frictionless price equation parameters (8), including the one measuring price complementarities.

[21] Formally, if $\frac{(p_{t-1}-p_t)}{p_{t-1}} > 25\%$ and $p_{t+1} \geq p_t\left(1+\frac{p_{t-1}-p_t}{p_{t-1}}\right)$. One can always say that 25% is an arbitrary value, and/or that it is high for some sectors and/or low for some others. But we decided to keep a single rule to minimize arbitrariness. Changing this value does not significantly change our main results.

[22] Anderson et al. (2012) using a very detailed database from a large retailer, where sales were explicity identified, concluded that this and other sales filter may bias some results. However, the proportion of sales is much smaller in our sample, as compared to US samples. Furthermore, our results do not change when a sales filter is not used.

We also estimate the variance of idionsyncratic shocks and the size of positive and negative adjustments.

5.1 The estimated models

The dependent variable is the observed $r_{i,t}$ defined in equation (16). Our baseline probit specification (18) has five explanatory variables and the whole set of dummy variables—one for each individual at each time, which are constructed according to equation (15). The first two variables are, respectively, the square root of the time interval between $\tau_{i,t}$ and t, and its inverse. They are obtained from the pricing history of each item in the microdata set. The last three variables are accumulated change since the last price adjustment (at time $\tau_{i,t}$) in the aggregate price level, $\Delta p_{i,t} = \log(P_t/P_{\tau_{i,t}})$, in the nominal expenditure, $\Delta \mathcal{Y}_{i,t} = \log(\mathcal{Y}_t/\mathcal{Y}_{\tau_{i,t}})$, and in the exchange rate, $\Delta e_{i,t} = \log(E_t/E_{\tau_{i,t}})$, all divided by the square root of the time elapsed since $\tau_{i,t}$, $\sqrt{\delta_{i,t}}$. As mentioned above, it is exactly this difference in the length of price spells what allows us to estimate the model using basically aggregate variables as regressors. Once firms reprice at different points in time, the accumulated changes in those variables since the last adjustment will be different for firms with different adjustment times. The denominator $\sqrt{\delta_{i,t}}$ accounts for the heteroskedasticity that appears in the residuals. Only two items with the same price adjustment history will have the same regressors.

In the baseline specification idiosyncratic shocks are assumed to follow a random walk, and the whole sample period is used in the estimation. For robustness, we also estimate the probit model changing some of the assumptions. We alternatively use a white-noise specification for the idiosyncratic shock, break the sample in two different subperiods, consider a closed economy version by excluding the exchange rate variable, and use different data treatments by not excluding temporary sales or not making itens exclusions due to problematic price quotes.

The measure of nominal expenditure we use in our estimations is the monthly nominal GDP series calculated by the Central Bank of Brazil using the quarterly national accounts of the Brazilian Institute of Geography and Statistics – IBGE. Our measure of aggregate price is the Consumer Price Index – IPC-FGV, which is calculated by the FGV from the set of microdata described in the previous section. The exchange rate is the monthly Real/US$ exchange rate (for sale) series provided by the Central Bank of Brazil.

5.2 Baseline specification

We now proceed to estimate the baseline specification of the probit model, including the degree of strategic complementarity in firm's price decisions, according to the identification strategy outlined in subsection 3.1. Table 2 reports the estimated parameters of the model.[23]

The parameters of the probit model are all significant at any of the usual significance levels. They also have the expected signs: nominal expenditure, the average price and the real exchange rate all affect positively the frictionless optimal price. This result is analagous to the one obtained by Barros et al. (2010)

[23] Detailed results of the probit model are presented in Appendix C.

using the same set of microdata by simply regressing the frequency of price changes on macro variables: the frequency of price change is significantly and positively related to inflation, output and exchange rate depreciation. In a state-dependent pricing model this connection should result from the frictionless optimal price relation with those same variables, and this is what our probit results are capturing.

We infer the parameter of strategic complementarity ζ from the relationship between the frictionless optimal price and the macroeconomic variables. The mapping between the probit model parameters and ζ is given by equation (21). Table 2 reports point estimates for ζ, as well as a 95% confidence interval obtained using the Delta method.

The estimated value of ζ is 0.06, which is substantially smaller than one. This implies that firms' pricing decisions are strongly complementary. This conclusion is also statistically warranted, since the upper bound of the confidence interval (0.07) is far below 1. In fact, it is even smaller than the value used in the Gertler and Leahy (2008) calibration of their state-dependent model. Because we do not have a complete model of the economy, we cannot simulate macro effects but, based on this comparison, we can argue that the magnitude of strategic complementarity we found in our data has the potential to produce substantial monetary policy effects in a state-dependent model.

Table 2: Probit parameters and strategic complementarity, baseline specification

$\mathcal{Y}_t$	p_t	e_t	ζ	Conf. interval for ζ	
0.60 (0.077)	9.57 (0.049)	0.11 (0.049)	0.06 (0.012)	0.05	0.07

Notes: The robust standard deviations are in parenthesis. The confidence interval is 95% of confidence. The standard deviation of ζ was obtained by the Delta method.

5.2.1 The estimated pricing rules

Now we explore the implications for the pricing rule of the probit model estimates. Table 3 shows the estimated parameters using the baseline specification. As can be seen, the intercepts[24] π_1 and π_0 are statistically significant at 1%.[25] Since we are able to estimate the parameter σ, from the intercepts of the probit model we can estimate the size of upward and downward adjustments using the pairwise distances between S, s and c calculated from the equations (23)—even though we cannot isolate S, s and c, individually. These results are reported in Table 3.

The size of downward and upward adjustments was estimated in 9% and 6%, respectively. This asymmetry, which is in accordance with direct evidence for the US and Brazil, is usually explained by the asymmetry of the profit function, but can also be additionally justified by the positive average inflation in Brazil during the period. However, the magnitude of those adjustments are smaller than the average found in the sample.

[24] We are calling them intercepts, but observe that our probit is not standard and the variable $1/\sqrt{\delta_{i,t}}$ changes over time.

[25] One fact that may help to explain this large significance is that we take into account the variability in the price spells duration—observe that all variables in the probit model are divided by the square root of the time interval between t and τ, $\sqrt{\delta_{i,t}}$. Objectively, this means that the intercepts are changing over time, which may help to improve the fit of the model.

Notice that the estimation did not use the size of price adjustments, and was based only a discrete variable that coded the ocurrence of price increases, price decreases or no adjustments. Due to the non-linearity in the relation between frequency and size of price adjustments (see Barros et al, 2010), the heterogeneity in the frequency of adjustments generates a downward bias in the estimated adjustment size.[26]

Finally, the standard deviation of the idiosyncratic shocks is estimated in 10%, which coincides with the value estimated by Klenow and Willis (2006) in the model without strategic complementarity.

Table 3: Estimated parameters of the pricing rule, baseline specification

$\pi_0 = \frac{c-S}{\sigma}$	$\pi_1 = \frac{c-s}{\sigma}$	Std. deviation σ	Size of upward adj. $c-s$	Size of downward adj. $S-c$	Size of inaction range
−0.92 (0.01)	0.62 (0.01)	0.10 (0.01)	0.06	0.09	0.15

5.3 Robustness

We also carried out a number of estimations for robustness check. First, we split the sample in two time periods: from 1997 to 2000 and from 2001 to 2004. The first and second rows of Table 4 display the results, that confirm the strong price complementarities found in the baseline estimation—the estimated ζ is 0.03 in the 1997-2000 period and 0.10 in the 2001-2004 period.[27]

We show in the third row an estimate of a closed economy version of the model, which does not include the real exchange rate in the frictionless optimal price equation. This formulation can be obtained from our structural model if we assume that labor is the only input in the production function. The estimated ζ increases substantially to 0.46, but prices are still strategic complements according to this measure. We view this specification with caution, since it is well known that exchange rate variation is a very important determinant of price inflation in Brazil.

In the baseline specification we estimated the model under the assumption that idiosyncratic shocks are random walk. But one could argue that idiosyncratic shocks are more transient than aggregate shocks and that the random walk assumption may bias our results. In order to check the robustness of our results to this assumption we reestimate the model under the alternative, and still treatable, assumption that idiosyncratic shocks are white noise. The results are shown in the fourth row of Table 4. The value obtained for ζ under this alternative is 0.07, still very close to the baseline specification.

There is also considerable debate in the liteof price dynamics. Thus, we reestimate our model with a data set not treated for temporary sales. The estimate for ζ was identicrature about whether one should

[26] Less frequent adjustments are associated with larger adjustments. The convexity of this relation implies that the use of the average frequency leads to the underestimation of the average size.

[27] We also estimated the model using data of each year separately during the whole sample period. Results are not reported, but they are also consistent with those already presented.

or not treat the temporary price decreases (sales or promotions) when working on models al to the one we obtained when promotions were treated.

Finally, we also estimated a model without excluding the itens with potential problematic price quotes, as raised by Eichenbaum, Jaimovich, Rebelo and Smith (2012). The last row of Table 4 reports the results, that still display considerable degree of price complementarities.

In summary, considering all variants of the open economy model we estimated, the point estimates of ζ were between 0.03 and 0.10, showing that our results are robust to those variations.

Table 4: Probit parameters and strategic complementarity (Robustness)

Model	$\mathcal{Y}_t$	p_t	e_t	ζ	Conf. interval for ζ	
Period 1997 - 2000	0.15 (0.160)	4.93 (0.146)	0.62 (0.024)	0.03 (0.031)	-0.03	0.09
Period 2001 - 2004	1.29 (0.094)	11.07 (0.054)	-0.40 (0.015)	0.10 (0.007)	0.09	0.12
Without Exchange Rate	2.93 (0.074)	3.43 (0.037)	$-$	0.46 (0.008)	0.44	0.48
Idiosyncratic White Noise	0.90 (0.112)	12.72 (0.072)	0.05 (0.018)	0.07 (0.008)	0.05	0.09
Without promotions	0.53 (0.079)	8.90 (0.050)	0.02 (0.013)	0.06 (0.008)	0.04	0.07
Without itens exclusions	0.44 (0.065)	9.79 (0.044)	0.03 (0.011)	0.04 (0.006)	0.03	0.05

Notes: The robust standard deviations are in parenthesis. The confidence interval is 95% of confidence. The standard deviation of ζ was obtained by the Delta method.

Alternative results for the pricing rule are also reported in Appendix C. In general they change very little. In particular, the similar results obtained for both 1997-2000 and 2001-2004 subperiods indicate that the assumption of a constant pricing policy is not too restrictive. The only important variation in the estimated pricing rules was found when idiosyncratic shocks were assumed to be white noise. In this case, both upward and downward adjustment sizes increased (to 8.2% and 11.9%, respectively), becoming closer to the actual adjustment sizes.

5.4 Actual, frictionless and reset price inflation

As a result of our estimation methodology, we are able to recover the *frictionless optimal price inflation* process, which is the price inflation that would occur if there were no price ridigities in the economy. We compare this process with the actual price inflation process. One would expect inflation to be more persistent than its frictionless counterpart. Our results confirm this intuition.

When pricing rules are Ss, as we assume, our frictionless optimal price inflation is also equivalent to the theoretical reset price inflation concept proposed by Bils, Klenow and Malin (2009, 2012) (hereafter BKM 2009 and BKM 2012). They calculate an empirical counterpart of reset price inflation based on individual price quotes, which is quite different from the theoretical one when rules are state-dependent. For comparison purposes we reproduce their methodology and also calculate their empirical reset price inflation measure in our sample. Thus, we are able to contrast statistical properties of empirical and theoretical reset price inflation process, and of those processes with the actual price inflation.

5.4.1 Reset price inflation

BKM constructed their measure with motivation similar to ours, in the sense that they aim to isolate factors that affect individual price from the frequency of price changes. They define *theoretical reset price* for an individual seller as that price which the seller would choose if he/she implemented a price change in the current period. Observe that this takes into consideration that the changed price is likely to last for several periods. Therefore, it differs from the definition of frictionless optimal price $p^*_{i,t}$ we use.[28] The *theoretical reset price inflation*, π^*_t, is the weighted average change of all reset prices, including those of current price changers and non-changers alike.

Their empirical measure of reset price for an item i at time t, $\hat{p}^*_{i,t}$, is given by:

$$\hat{p}^*_{i,t} = \begin{cases} p_{i,t}, & \text{if } p_{i,t} \neq p_{i,t-1} \\ \hat{p}^*_{i,t-1} + \hat{\pi}^*_t, & \text{if } p_{i,t} = p_{i,t-1} \end{cases}, \tag{24}$$

where $\hat{\pi}^*_t$, the *reset price inflation*, is given by:

$$\widehat{\pi}^*_t \equiv \frac{\sum_i \omega_{i,t} \left(p_{i,t} - \hat{p}^*_{i,t-1}\right) I_{i,t}}{\sum_i \omega_{i,t} I_{i,t}},$$

where $I_{i,t}$ is equal to 1 if $p_{i,t} \neq p_{i,t-1}$ and zero otherwise.

Notice that since one cannot observe the reset price for those not changing prices, their empirical method updates reset prices for them using the reset inflation $\widehat{\pi}^*_t$. That measure, in turn, is defined by the weighted average of the difference between new prices in t and reset prices defined in $t-1$. As BKM acknowledge, firms that change price may have stronger incentive to do so, which means that there is a selection effect. As a consequence, the theoretical reset price inflation, which is based on desired prices for changers and non-changers, may differ markedly from its empirical counterpart when there is the selection effect. In our methodology the selection effect is taken into account by our assumed state-dependent pricing policy through the probability of price change estimated by the probit model. Price adjustments occur when prices are distant enough from their frictionless optimal counterpart.

Their method of investigation is to use a general equilibrium price-setting model to simulate individual price data and then compare the behavior of reset price inflation from the model to the one constructed with the microdata underpinning the U.S. CPI. BKM construct several statistics based on their empirical measure of reset price inflation to evaluate different macro models. In their working paper version (BKM

[28] The two measures differ by a constant if pricing rules are two-sided Ss, as we assume in this paper.

2009), they use them to discriminate among time or state-dependent models of price-setting, with and without strategic complementarities.[29] In their baseline cases, they close the model with an exogenous process for the money supply. Then they simulate the model and compute the standard deviation and serial correlation of reset price inflation and actual inflation. They also show impulse response functions for reset price inflation with base on univariate AR(6) regressions. They conclude that the state-dependent model without strategic complementarity fits better the proposed set of statistics than the alternative models. The model with stategic complementarities is rejected since the reset price inflation generated by the simulated model displays higher persistence and lower volatility than the ones based on the actual data.[30] Since the reason for the difference between our findings and their findings could be that we use Brazilian microdata and they use US microdata, we repeat their experiment with our data.[31]

Selection

5.4.2 Frictionless optimal price inflation

The comparison of the statistical properties of the BKM's reset price inflation to the properties of our frictionless optimal inflation can be very informative. If all selection effect is adequately taken care by our Ss pricing model, strategic complementarities should make frictionless optimal inflation more persistent and stable. On the other hand, empirical reset price inflation persistence is influenced both by the persistence of the frictionless optimal inflation and the selection effect. The latter effect in isolation tend to induce a negative correlation in reset price inflation, making empirical reset price inflation less persistent than the frictionless optimal inflation.

In order to construct an empirical measure for aggregate frictionless optimal price inflation, we aggregate individual frictionless optimal price inflation, using the results of the probit model in the baseline specification:

$$\pi_t^* = \sum_i \omega_{i,t} \pi_{i,t}^*, \tag{25}$$

where $\omega_{i,t}$ is the weight of each item in the CPI-FGV, and the individual frictionless optimal price inflation for item i is estimated by

$$\begin{aligned} \pi_{i,t}^* &= p_{i,t}^* - p_{i,t-1}^* = \Delta \boldsymbol{x}_t' \hat{\boldsymbol{\beta}} + (\hat{\hat{\xi}}_t - \hat{\hat{\xi}}_{t-1}) + (\hat{\hat{a}}_{i,t} - \hat{\hat{a}}_{i,t-1}) \\ &= \hat{\eta} + \Delta \boldsymbol{x}_t' \hat{\boldsymbol{\beta}} + (\hat{\hat{\xi}}_t - \hat{\hat{\xi}}_{t-1}) + \hat{\varepsilon}_{i,t}, \end{aligned} \tag{26}$$

[29] The focus of their published article (BKM 2012) is the evaluation of Smets and Wouters (2007) model.

[30] They also found that TDP models with or without strategic complementarity generate reset price inflation series that are too persistent.

[31] We do not have access to disaggregated data on prices collected by the U.S. Bureau of Labor Statistics. Otherwise, we could use these data in our estimations.

where $\hat{\boldsymbol{\beta}}$, $\hat{\eta}$ and $\hat{\hat{\xi}}_t$[32] are the parameters of the probit model estimated in the baseline specification. To construct a measure of frictionless optimal price inflation we carry out a Monte Carlo experiment. For each individual item in our data set we simulate one trajectory for $\{\hat{\varepsilon}_{i,t}\}_{t=1}^{T}$ using the distribution $\mathbb{N}(0, \hat{\sigma}^2)$, where $\hat{\sigma}^2$ is the estimated variance of the idiosyncratic shocks. We then aggregate the individual frictionless optimal price inflations from (26) using equation (25), to have π_t^*. We repeat this experiment one thousand times and compare the statistical properties of these series to those of the reset price inflation.

5.4.3 Actual, frictionless and reset price inflation statistics

Figure 1 plots the series for frictionless optimal, reset, and actual price inflation. It is immediately apparent from the figure that reset price inflation has much larger variability and lower persistence than both frictionless and actual inflation. Table 5 reports the average, the standard deviation and the serial correlation for the three variables. The frictionless inflation has autocorrelation coefficient of 0.21, being less persistence than actual inflation (autocorrelation of 0.51), but substantially more persistent than the reset price inflation. The selection effect seems to be the main driver of the negative autocorrelation (-0.35) and the large volatility of reset price inflation (2.30). The volatilty of the frictionless optimal inflation is much smaller (0.72), being of the same order of magnitude than that of actual inflation (0.76). These results do not change if we use seasonally adjusted series or not, as the last rows of Table (5) show.

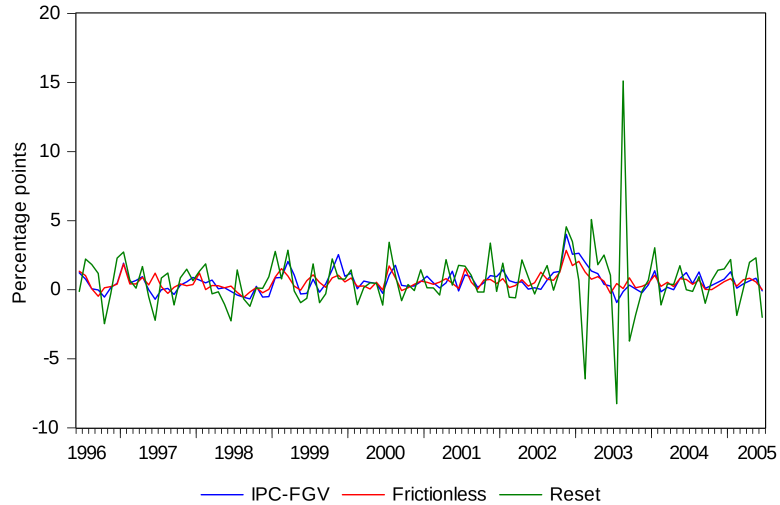
Table 5: Summary statistics for frictionless, reset and actual inflation after exclusions

Series	Average	Std deviation	Persistence
Actual Inflation CPI-FGV	0.57%	0.76%	0.51
Frictionless Inflation	0.52%	0.72%	0.21
Reset Price Inflation	0.62%	2.30%	-0.35
Actual Inflation CPI-FGV, Seas. Adj.	0.57%	0.69%	0.51
Frictionless Inflation, Seas. Adj.	0.52%	0.63%	0.26
Reset Price Inflation, Seas. Adj.	0.62%	2.07%	-0.41

Notes: 1) The statistics are calculated using the period Jun/1996-Aug/2005. 2) Frictionless inflation is obtained by aggregating $\boldsymbol{\pi}_{i,t}^*=\hat{\eta}+\Delta \boldsymbol{x}_t'\hat{\boldsymbol{\beta}}+(\hat{\hat{\xi}}_t-\hat{\hat{\xi}}_{t-1})+\hat{\varepsilon}_{i,t}$, where $\hat{\eta}$ and $\hat{\boldsymbol{\beta}}$ are the estimated parameters in the baseline model. Aggregation uses the good weights in the IPC-FGV. The terms $\hat{\varepsilon}_{i,t}$ come from a Monte Carlo experiment with 1000 simulation. 3) Persistence is measuredby the first-order autocorrelation. For frictionless inflation, average, standard deviation and persistence are the averages across the values obtained in the simulation.

We also follow BKM in producing impulse response functions (IRF) with base on univariate AR(6) regressions. Figures 2, 3 and 4, respectively, display the impulse response function (IRF) (in level) for actual, frictionless and for reset price inflation in our data. Figure 3 plots the average response of frictionless price and a fan-chart with various levels of significance obtained in the simulation. It shows that, after a shock, frictionless optimal price has an upward sloping trajectory. The shape is similar to the impulse response for our actual inflation, in Figure 2. The strong price complementarities we found through our

[32] $\hat{\hat{\xi}}_t$ is given by the coefficient of the dummy for the aggregate shock at time t.



Note: Frictionless inflation corresponds to the average obtained across the 1000 simulations. Actual inflation is calculated among products left after exclusions. Reset inflation is obtained using BKM's methodology.

Figure 1: Frictionless, reset and actual price inflation

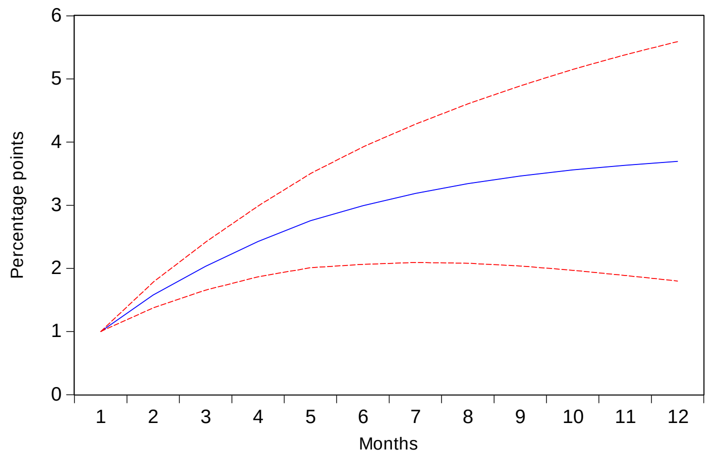


Figure 2: Impulse response of actual inflation

estimation is also consistent with the frictionless optimal price response: the impact is initially small (as price setters wait for the average price to respond), but accumulates over time as more firms change prices. This pattern is also similar to that found by BKM 2009 for the impulse response of *theoretical* reset prices generated by their state-dependent model with strategic complementarities. Since their theoretical reset price correspond to our frictionless optimal level, this indicates that our estimates are consistent with their baseline state-dependent model with strategic complementarities.[33]

Notice that statistics based on the empirical reset price inflation in our sample are similar to those reported by BKM 2009 for the US. Interestingly, their simulated state-dependent model without strategic complementarity generates a standard deviation of 1.79% and a serial correlation of -0.31, fitting our data set even better than theirs. Their simulated model also fits our inflation statistics better than the US statistics, although not as close as the reset price inflation. Visual inspection of the IRF for BKM 2009 simulated model reinforces the conclusion that their state-dependent model without strategic complementarity fits reset price inflation from our data base even more closely than theirs. Figure 4 shows that the response of reset prices is much greater on impact than over time, exactly like those in BKM 2009. Therefore, if we were to use the same methodology as them in our data set, we would arrive at that same conclusion: the state-dependent model without strategic complementarities fits better the data than the state-dependent model with strategic complementarity.

If it is not the different data set, what could be the reason for the different conclusions? The difference

[33] In their baseline case, they close the model with an exogenous money suply rule.

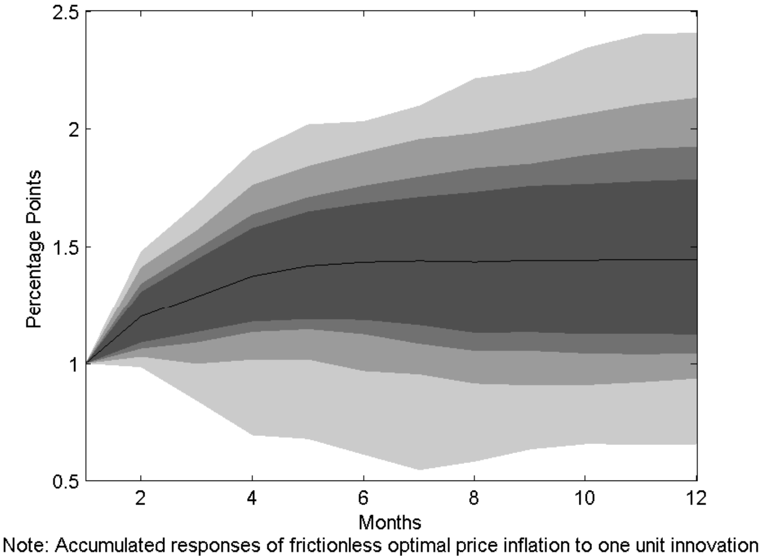


Figure 3: Impulse response of estimated frictionless optimal price, baseline specification

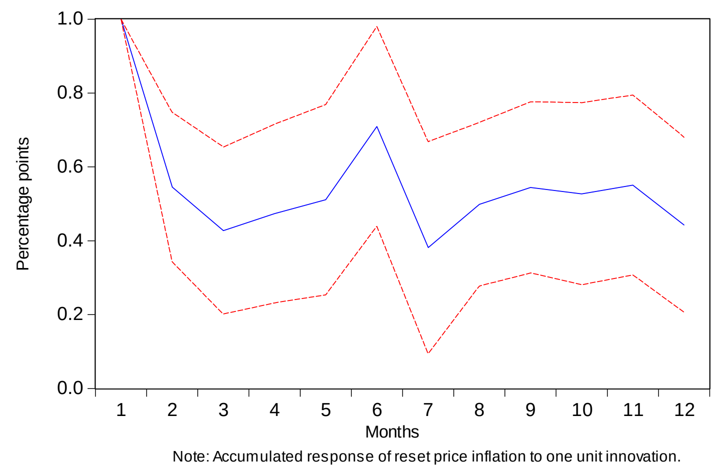


Figure 4: Impulse response of estimated reset price, using BKM's methodology

must come from methodology. Their exercise is based on model simulation. Although their focus in on the supply side—price setting and strategic complementarity—, in order to simulate the model they must close it with a demand side. As their last exercise shows, the specification of the demand side can make a big difference: when they allowed money supply to respond to real aggregate demand, the state-dependent model with strategic complementarity turned out to fit better the reset price inflation IRS than the model without strategic complementarity.[34] This result suggests that in simulated models the demand side specification, particularly the modeling of monetary policy, could be as important as the supply side.

By contrast, our methodology focus on estimating the frictionless optimal price equation. The parameters of this equation do not depend on the frequency of price adjustments or on the demand side specification. We also control for the selection of price changes through our assumed state-dependent pricing rule. Of course, our method could be subject to other problems as, for example, a misspecification of pricing rule.

6 Conclusions

In this paper we have developed a methodology to directly estimate the parameter measuring the degree of strategic complementarities in pricing decisions in a state-dependent pricing model. From an equation of firm's frictionless optimal price which can be obtained from a variety of structural models, and an assumed Ss pricing rule, we derive a structural, non-standard ordered probit model. The assumed pricing rule makes the probability of adjustments in such a model to be governed by changes in the frictionless optimal price since the last adjustment. We estimate the model using a quasi-maximum likelihood method that takes into account the emerging autocorrelation and the heteroskedasticity with base on the microdata underpinning the CPI-FGV in Brazil for the 1996-2006 period.

We find a substantial degree of strategic complementarity in firms' pricing decisions. Our estimates for the parameter ζ measuring strategic complementarities range from 0.03 to 0.10. Therefore, our results contrast with those of Bills, Klenow and Malin (2009), who suggested that a state-dependent pricing model with strategic complementarities is in contradiction with the data. We applied their methodololgy to our data and found results that are similar to theirs. The difference, thus, must come from the methodology, and not from the different data set. In particular, a more thorough examination of role of the demand side on the dynamics of a complete model should help to clarify the source of the differences.

A limitation of our results is that although they seem to be robust to the specification of the demand side, they are not certainly robust to the type of pricing rule. If the right model has some time dependency due to information frictions, strategic complementarities found in the estimated state-dependent model could be a compensation for the mispecification in the type of pricing rule. Thus, a natural next step should be to develop a similar methodology for a more general class of pricing models that include both time- and

[34] They do not select this model as the best because it generates a inflation rate process that is too smooth.

state-dependency (e.g. Bonomo, Carvalho and Garcia 2011), to see how sensitive are the degree of strategic complementarities estimates to information costs.

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A Derivation of real marginal cost equation

The optimal quantities of labor and foreign input required to produce the quantity $Y_{i,t}$ of good i is given by

$$L_{i,t} = \left(\frac{\alpha}{(1-\alpha)} \frac{\tilde{E}_t}{W_{it}} \right)^{1-\alpha} \frac{Y_{i,t}}{A_{i,t}}, \tag{27}$$

$$M_t = \left(\frac{(1-\alpha)}{\alpha} \frac{W_{i,t}}{\tilde{E}_t} \right)^{\alpha} \frac{Y_{i,t}}{A_{i,t}}. \tag{28}$$

Then, the real cost of supplying the quantity $Y_{i,t}$ is

$$\begin{aligned} w_{i,t}L_{i,t} + E_t M_t &= \alpha^{-\alpha}(1-\alpha)^{(\alpha-1)} \frac{w_{i,tt}^{\alpha} E_t^{(1-\alpha)} Y_{i,t}}{A_{i,t}} \\ &= \lambda \frac{w_{i,t}^{\alpha} E_t^{(1-\alpha)} Y_{i,t}}{A_{i,t}}, \end{aligned} \tag{29}$$

where $\lambda = \alpha^{-\alpha}(1-\alpha)^{(\alpha-1)}$, $w_{i,t}$ is the real wage of type i labor, and E_t is the real exchange rate.

Since we assume that each labor market is competitive, each firm producing variety of type i, take type i wage as given when choosing how much to produce. Thus, its marginal cost is given by:

$$\lambda \frac{w_{i,t}^{\alpha} E_t^{(1-\alpha)}}{A_{i,t}}$$

In order to eliminate $w_{i,t}$ from the marginal cost equation, first note that by equation (4), the quantity of labor is positively related to the firm's product. Therefore, we will rewrite equation (4) as

$$w_{i,t} = \frac{V_{i,t} L_{i,t}^{\delta}}{U_t C_t^{-\frac{1}{\sigma}}} = \frac{V_{i,t} \left[\left(\frac{\alpha}{(1-\alpha)} \frac{E_t}{w_{i,t}} \right)^{1-\alpha} \frac{Y_{i,t}}{A_{i,t}} \right]^{\delta}}{U_t Y_t^{-\frac{1}{\sigma}}} \tag{30}$$

Then, we can write the real wage of type i as

$$\begin{aligned} w_{i,t}^{1+\delta(1-\alpha)} &= \left[\frac{(1-\alpha)}{\alpha} \right]^{\delta(1-\alpha)} \left(\frac{Y_{it}}{A_{i,t}} \right)^{\delta} Y_t^{\frac{1}{\sigma}} E_t^{\delta(1-\alpha)} V_{i,t} U_t^{-1} \\ w_{i,t} &= \left[\frac{(1-\alpha)}{\alpha} \right]^{\frac{\alpha\delta(1-\alpha)}{1+\delta(1-\alpha)}} \left(\frac{Y_{it}}{A_{i,t}} \right)^{\frac{\delta}{1+\delta(1-\alpha)}} Y_t^{\frac{1}{\sigma(1+\delta(1-\alpha))}} E_t^{\frac{\delta(1-\alpha)}{1+\delta(1-\alpha)}} V_{i,t}^{\frac{1}{1+\delta(1-\alpha)}} U_t^{-\frac{1}{1+\delta(1-\alpha)}} \end{aligned}$$

Substituting the real wage expression into marginal cost equation yields:

$$
\begin{aligned}
\psi(Y_{i,t}, Y_t, E_t; V_{i,t}, U_t, A_{i,t}) &= \lambda \frac{w_{i,t}^{\alpha} E_t^{(1-\alpha)}}{A_{i,t}} = \frac{\lambda Y_{i,t}}{A_{i,t}} \left[\frac{(1-\alpha)}{\alpha}\right]^{\frac{\alpha^2 \delta(1-\alpha)}{1+\delta(1-\alpha)}} \left(\frac{Y_{it}}{A_{i,t}}\right)^{\frac{\alpha\delta}{1+\delta(1-\alpha)}} \\
& \quad Y_t^{\frac{\alpha}{\sigma(1+\delta(1-\alpha))}} E_t^{\frac{\alpha\delta(1-\alpha)}{1+\delta(1-\alpha)}} E_t^{(1-\alpha)} V_{i,t}^{\frac{\alpha}{1+\delta(1-\alpha)}} U_t^{-\frac{\alpha}{1+\delta(1-\alpha)}} \\
&= \lambda \left[\frac{(1-\alpha)}{\alpha}\right]^{\frac{\alpha^2 \delta(1-\alpha)}{1+\delta(1-\alpha)}} A_{i,t}^{-\frac{1+\delta}{1+\delta(1-\alpha)}} Y_t^{\frac{\alpha}{\sigma(1+\delta(1-\alpha))}} Y_{it}^{\frac{\alpha\delta}{1+\delta(1-\alpha)}} E_t^{\frac{(1-\alpha)+\delta(1-\alpha)}{1+\delta(1-\alpha)}} \\
& \quad V_{i,t}^{\frac{\alpha}{1+\delta(1-\alpha)}} U_t^{-\frac{\alpha}{1+\delta(1-\alpha)}}
\end{aligned}
$$

B Econometric Estimation

In this appendix we discuss the econometric estimation of the empirical model proposed in this paper and we provide asymptotic results when the number of cross-section units (N) tends to infinity and the time series dimension (T) is fixed. Considering T fixed is important in order to avoid the incidental parameter problem. This is not a strong restriction as in our application N is much larger than T.

B.1 The likelihood function

For notational reason, write equations in (18) in the following simpler way:

$$\begin{aligned} Pr[r_{i,t}=1|\boldsymbol{w}_{i,t}] &= 1-\Phi\left(\pi_1\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right) \\ Pr[r_{i,t}=0|\boldsymbol{w}_{i,t}] &= \Phi\left(\pi_1\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right)-\Phi\left(\pi_0\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right), \\ Pr[r_{i,t}=-1|\boldsymbol{w}_{i,t}] &= \Phi\left(\pi_0\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right), \end{aligned}$$

where $\ddot{\boldsymbol{w}}_{i,t}=(\ddot{\delta}_{i,t},\ddot{\boldsymbol{z}}'_{i,t},\ddot{\boldsymbol{d}}'_{i,t})'$ and $\tilde{\boldsymbol{\theta}}=\left(\tilde{\eta},\tilde{\boldsymbol{\beta}}',\tilde{\gamma}_1,\ldots,\tilde{\gamma}_T\right)'$.

The partial log-likelihood of an observation i at time t is given by:

$$\begin{aligned} \ell_{i,t}(\boldsymbol{\psi}) = {} & \mathbb{I}\{r_{i,t}=1\}\log\left[1-\Phi\left(\pi_1\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right)\right]+ \\ & +\mathbb{I}\{r_{i,t}=0\}\log\left[\Phi\left(\pi_1\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right)-\Phi\left(\pi_0\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right)\right]+ \\ & +\mathbb{I}\{r_{i,t}=-1\}\log\left[\Phi\left(\pi_0\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right)\right], \end{aligned}$$

where $\boldsymbol{\psi}=(\pi_1,\pi_0,\tilde{\boldsymbol{\theta}}')'$ and $\mathbb{I}\{\cdot\}$ is the indicator function.

Define

$$\begin{aligned} \Phi_1(.) &\equiv \Phi\left(\pi_1\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right), \\ \Phi_0(.) &\equiv \Phi\left(\pi_0\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right), \\ \phi_1(.) &\equiv \phi\left(\pi_1\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right), \\ \phi_0(.) &\equiv \phi\left(\pi_0\ddot{\imath}_{i,t}-\ddot{\boldsymbol{w}}'_{i,t}\tilde{\boldsymbol{\theta}}\right). \end{aligned}$$

Therefore, the score of an observation i at time t is given by:

$$\boldsymbol{s}_{i,t}(\boldsymbol{\psi})=\begin{bmatrix}\frac{\partial\ell_{i,t}}{\partial\pi_1}\\ \frac{\partial\ell_{i,t}}{\partial\pi_0}\\ \frac{\partial\ell_{i,t}}{\partial\boldsymbol{\theta}}\end{bmatrix}=\begin{bmatrix}s_{1,i,t}\\ s_{2,i,t}\\ \boldsymbol{s}_{3,i,t}\end{bmatrix},$$

where

$$s_{1,i,t} = \left[\frac{\mathbb{I}\{r_{i,t}=0\}}{\Phi_1(.)-\Phi_0(.)} - \frac{\mathbb{I}\{r_{i,t}=1\}}{1-\Phi_1(.)} \right] \phi_1(.)\ddot{\imath}_{i,t},$$

$$s_{2,i,t} = \left[\frac{\mathbb{I}\{r_{i,t}=-1\}}{\Phi_0(.)} - \frac{\mathbb{I}\{r_{i,t}=0\}}{\Phi_1(.)-\Phi_0(.)} \right] \phi_0(.)\ddot{\imath}_{i,t},$$

$$\boldsymbol{s}_{3,i,t} = \left\{ \frac{\mathbb{I}\{r_{i,t}=1\}\phi_1(.)}{1-\Phi_1(.)} + \frac{\mathbb{I}\{r_{i,t}=0\}}{\Phi_1(.)-\Phi_0(.)}\left[-\phi_1(.)+\phi_0(.)\right] - \frac{\mathbb{I}\{r_{i,t}=-1\}\phi_0(.)}{\Phi_0(.)} \right\} \tilde{\boldsymbol{w}}_{i,t}.$$

Furthermore, define the following quantities:

$$\ell_i = \sum_{t=1}^{T} \ell_{it},$$

$$\boldsymbol{s}_i(\boldsymbol{\psi}) = \sum_{t=1}^{T} \boldsymbol{s}_{it}(\boldsymbol{\psi}),$$

$$\boldsymbol{A}_0 = -\mathbb{E}\left[\left. \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \boldsymbol{\psi} \partial \boldsymbol{\psi}'} \right|_{\boldsymbol{\psi}=\boldsymbol{\psi}_0} \right] = -\sum_{t=1}^{T} \mathbb{E}\left[\left. \frac{\partial^2 \ell_{it}(\boldsymbol{\psi})}{\partial \boldsymbol{\psi} \partial \boldsymbol{\psi}'} \right|_{\boldsymbol{\psi}=\boldsymbol{\psi}_0} \right], \text{ and}$$

$$\boldsymbol{B}_0 = \mathbb{E}\left\{ \left[\boldsymbol{s}_i(\boldsymbol{\psi}_0)\right]\left[\boldsymbol{s}_i(\boldsymbol{\psi}_0)\right]' \right\} = \mathbb{E}\left\{ \left[\sum_{t=1}^{T} \boldsymbol{s}_{it}(\boldsymbol{\psi}_0) \right] \left[\sum_{t=1}^{T} \boldsymbol{s}_{it}(\boldsymbol{\psi}_0) \right]' \right\}.$$

B.2 Main assumptions and asymptotic results

Consider the following set of assumptions.

Assumption 1 *The frictionless price follows the model*

$$p^*_{i,t} = \kappa + \zeta \mathcal{Y}_t + (1-\zeta)p_t + \frac{(1+\delta)(1-\alpha)}{1+\delta(1-\alpha)+\theta\delta\alpha} e_t + \tilde{\xi}_t + \tilde{a}_{i,t}, \tag{31}$$

where $\mathcal{Y}_t$ is the nominal expenditure, p_t is the aggregated price index, κ and ζ are combinations of deep parameters of the economy, $\tilde{\xi}_t$ is the aggregate shock and $\tilde{a}_{i,t}$ is the idiosyncratic shock.

Assumption 2 *The aggregated shock $\tilde{\xi}_t$ follows a random walk process*

$$\tilde{\xi}_t = \tilde{\xi}_{t-1} + v_t,\ t = 1, \dots, T,$$

where $\tilde{\xi}_0 = O_p(1)$, $\mathbb{E}(v_t) = 0$, $\mathbb{E}\left(|v_t|^a\right) < \infty$, for $a = 1, \dots, 4$. Finally, defining $\boldsymbol{v} = (v_1, \dots, v_T)'$, $\mathbb{E}(\boldsymbol{v}\boldsymbol{v}')$ is a $T \times T$ positive definite matrix.

Assumption 3 *For each individual i, the idiosyncratic shocks, $\tilde{a}_{i,t}$, follows one of the following processes:*

1. $\tilde{a}_{i,t} = a_i + \tilde{a}_{i,t-1} + \varepsilon_{i,t}$, *where* $\tilde{\xi}_0 = O_p(1)$; *or*
2. $\tilde{a}_{i,t} = a_i + \varepsilon_{i,t}$.

In both cases a_i is a fixed effect and $\varepsilon_{i,0} = O_p(1)$, $\mathbb{E}(\varepsilon_{i,t}) = 0$, $\mathbb{E}\left(|\varepsilon_{i,t}|^a\right) < \infty$, for $a = 1, \ldots, 4$. Furthermore, $\mathbb{E}(\varepsilon_{i,t} v_\tau) = 0$, $\forall\, i, \tau$. Finally, defining $\boldsymbol{\varepsilon}_i = (\varepsilon_{i,1}, \ldots, \varepsilon_{i,T})'$, $\mathbb{E}\left(\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_i'\right)$ is a $T \times T$ positive definite matrix.

Assumption 4 *The log of the aggregate price index, p_t, is given by the following weighted sum:*

$$p_t = \sum_{n=1}^{N} \omega_{n,t} p_{n,t}, \tag{32}$$

where $p_{n,t}$ is the individual log price index and $0 < \omega_{n,t} < 1$ is a weight such that

$$\omega_{n,t} \longrightarrow 0 \; as \, N \longrightarrow \infty, \forall\, t. \tag{33}$$

Assumption 5 *The variables $\{\boldsymbol{q}_{it}\} = \left\{\left(\ddot{1}_{it}, \ddot{\boldsymbol{z}}_{it}'\right)'\right\}$ are uniformly bounded in i and $\lim_{N\to\infty} N^{-1} \sum_{t=1}^{T} \sum_{i=1}^{N} \boldsymbol{q}_{it} \boldsymbol{q}_{it}'$ is a finite nonsingular matrix.*

Assumption 6 *The true parameter vector $\boldsymbol{\psi}_0$ is in the interior of $\boldsymbol{\Psi}$, a compact parameter space.*

Assumption 7 *The variable $\delta_{i,t} \equiv (t - \tau_{it})$ is such that, for each $t = 1, \ldots, T$:*

$$\lim_{N\to\infty} \frac{1}{N} \mathbb{E}\left[\sum_{i=1}^{N} \left(\delta_{i,t} - \overline{\delta}_t\right)^2\right] \to \Delta_t > 0,$$

where $\overline{\delta}_t = \frac{1}{N} \sum_{i=1}^{N} \delta_{i,t}$ and Δ_t is a non-stochastic constant. Furthermore, $\Delta_t < \infty$.

Assumptions 1–3 refer to the data generating process (DGP). More specifically, Assumption 1 describes the price evolution and Assumptions 2 and 3 determines the DGP for the aggregate and idiosyncratic shocks, respectively. Altogether, these three assumptions guarantees that we have a correctly specified model. We do not impose any specific autocorrelation among shocks with different time periods, just some moment conditions. Of course, the idiosyncratic shocks must be uncorrelated with the aggregate shock for consistency of the parameter estimates. Assumption 4 is important to guarantee that the aggregate price is an exogenous variable. Under 4, the relative contribution of a single product tends to zero as the number of products (cross-section units) tends to infinity. This is a reasonable assumption considering the structure of the data used in this paper. Assumption 5 is a common assumption in most of econometric models. It imposes that there is no multi-collinearity and no explosive variables. Assumption 6 is standard in the econometrics literature. Finally, 7 imposes that different firms change their prices at different times. This is the key assumption in order to identify the aggregate shock. This assumption guarantees that the set of dummy variables are not perfectly correlated with $\ddot{\boldsymbol{z}}_{it}$.

The quasi maximum likelihood estimator (QMLE) is defined as

$$\widehat{\boldsymbol{\psi}} = \underset{\boldsymbol{\psi}\in\boldsymbol{\Psi}}{\mathsf{argmax}} \sum_{i=1}^{N}\sum_{t=1}^{T} \ell_{i,t}(\boldsymbol{\psi}). \tag{34}$$

The next lemma presents a simple results showing that under the set of assumptions above, the idiosyncratic shock is exogenous.

Lemma 1 *Let $\varepsilon_{i,t}$ be defined as in Assumption 3. Under Assumption 4, $\mathbb{E}\left(\varepsilon_{i,t}p_t\right) \longrightarrow 0$ as $N \longrightarrow \infty$, for all t.*

Proof: Note that

$$\begin{aligned}\mathbb{E}\left(\varepsilon_{i,t}p_t\right) &= \mathbb{E}\left(\varepsilon_{it}\sum_{n=1}^{N}\omega_{n,t}p_{n,t}\right)\\ &= \omega_{i,t}\mathbb{E}(\varepsilon_{i,t}p_{i,t}).\end{aligned}$$

If $-\infty < \mathbb{E}(\varepsilon_{i,t}p_{i,t}) < \infty$, for all i and t, under Assumption 4, $\omega_{i,t}\mathbb{E}(\varepsilon_{i,t}p_{i,t}) \longrightarrow 0$ as $N \longrightarrow \infty$. ■

The next theorem is a straightforward result in quasi-maximum likelihood estimation and the proof follows standard reasoning.

Theorem 1 *Under Assumptions 1–7, $\widehat{\boldsymbol{\psi}} \xrightarrow{p} \boldsymbol{\psi}_0$ and*

$$\sqrt{N}\left(\widehat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right) \xrightarrow{d} \mathsf{N}\left(\mathbf{0}, \boldsymbol{\Omega}\right),$$

where $\boldsymbol{\Omega} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}^{-1}$. Furthermore, $\boldsymbol{\Omega}$ can be consistently estimated by $\widehat{\boldsymbol{\Omega}} = N^{-1}\widehat{\boldsymbol{A}}^{-1}\widehat{\boldsymbol{B}}\widehat{\boldsymbol{A}}^{-1}$, where

$$\widehat{\boldsymbol{A}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})'$$

and

$$\widehat{\boldsymbol{B}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})' + \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{r\neq t} \boldsymbol{s}_{i,r}(\hat{\boldsymbol{\psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})'.$$

B.3 Confidence interval for $\hat{\zeta}$ and for $\hat{\sigma}$

Theorem (Delta Method): Suppose that $\hat{\boldsymbol{\theta}}$ is an estimator of the Px1 vector $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and that

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} \mathbb{N}(\mathbf{0}, \boldsymbol{V}),$$

where $\boldsymbol{V}$ is a PxP positive definite matrix. Let $\boldsymbol{c} : \boldsymbol{\Theta} \to \mathbb{R}^Q$ be a continuous differentiable function on the parameter space $\boldsymbol{\Theta} \subset \mathbb{R}^P$, where $Q \leq P$, and assume that $\boldsymbol{\theta}$ is in the interior of the parameter space. Define $\boldsymbol{C}(\boldsymbol{\theta}) \equiv \nabla_\theta \boldsymbol{c}(\boldsymbol{\theta})$ and the QxP Jacobian of $\boldsymbol{c}$. Then

$$\sqrt{N}[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})] \xrightarrow{d} \mathbb{N}[\mathbf{0}, \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{V}\boldsymbol{C}(\boldsymbol{\theta})']. \tag{35}$$

Define $\hat{\boldsymbol{C}} \equiv \boldsymbol{C}(\hat{\boldsymbol{\theta}})$. Then plim $\hat{\boldsymbol{C}} = \boldsymbol{C}(\boldsymbol{\theta})$. If plim $\hat{\boldsymbol{V}} = \boldsymbol{V}$, then

$$N[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})]'[\hat{\boldsymbol{C}}\hat{\boldsymbol{V}}\hat{\boldsymbol{C}}']^{-1}[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})] \xrightarrow{d} \chi^2_Q. \tag{36}$$

Proof: See Wooldridge (2002), pp. 44-45 ■

Once ζ and σ are given respectively by $\zeta = \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}$ and $\sigma = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}$, we can use the theorem above to obtain

$$\sqrt{N}[\hat{\zeta} - \zeta] \xrightarrow{d} \mathbb{N}[0, \boldsymbol{C}_\zeta \boldsymbol{V}_1 \boldsymbol{C}'_\zeta] \tag{37}$$

and

$$\sqrt{N}[\hat{\sigma} - \sigma] \xrightarrow{d} \mathbb{N}[0, \boldsymbol{C}_\sigma \boldsymbol{V}_1 \boldsymbol{C}'_\sigma], \tag{38}$$

where $\boldsymbol{C}_\zeta = [C^1_\zeta \; C^2_\zeta]'$, $\boldsymbol{C}_\sigma = [C^1_\sigma \; C^2_\sigma]'$ and $C^1_\zeta = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2} - \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}$, $C^2_\zeta = -\frac{\tilde{\beta}_1}{(\tilde{\beta}_1 + \tilde{\beta}_2)^2}$, $C^1_\sigma = -\frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}$, $C^2_\sigma = -\frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}$. $\boldsymbol{V}_1$ is the respective variance-covariance matrix of $\tilde{\beta}_1$ and $\tilde{\beta}_2$.

Now the confidence interval for the desired significance level can be constructed as usual.

B.4 Derivation of the final equation for $r^*_{i,t}$

Under Assumption 1 we have:

$$r^*_{i,t} = p_{i,\tau_{i,t}} - p^*_{i,t} = c - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - (\tilde{\xi}_t - \tilde{\xi}_{\tau_{i,t}}) - (\tilde{a}_{i,t} - \tilde{a}_{i,\tau_{i,t}}).$$

First, consider the term $\tilde{\xi}_t - \tilde{\xi}_{\tau_{i,t}}$. Under Assumption 2 and iterating $\tilde{\xi}_t$ backward we obtain:

$$\tilde{\xi}_t - \tilde{\xi}_{\tau_{i,t}} = v_t + \cdots + v_{t-\delta_{i,t}+1}.$$

Empirically, we capture these aggregate shocks through dummy variables. Define one dymmy for each period t of time. Thus, we can write:

$$v_t = \sum_{j=1}^{T} \gamma_j d_t(j),$$

where

$$d_t(j) = \begin{cases} 1, & \text{if}\, t = j \\ 0, & \text{if}\, t \neq j \end{cases}.$$

Hence,

$$\begin{aligned} \tilde{\xi}_t - \tilde{\xi}_{\tau_{i,t}} =& v_t + v_{t-1} + \cdots + v_{t-\delta_{i,t}+1} \\ =& \sum_{j=1}^{T} \gamma_j d_t(j) + \sum_{j=1}^{T} \gamma_j d_{t-1}(j) + \cdots + \sum_{j=1}^{T} \gamma_j d_{j,t-\delta_{i,t}+1}(j) \\ =& \sum_{j=t-\delta_{i,t}+1}^{t} \gamma_j d_t(j) \\ =& \sum_{j=1}^{T} \gamma_j d_{i,t}(j), \end{aligned}$$

where

$$d_{i,t}(j) = \begin{cases} 1, & \text{if } j \in [t, t-\delta_{i,t}+1] \\ 0, & \text{if otherwise} \end{cases}.$$

The equality in the third line follows from the fact that, in each term of the sum, only one dummy lights up, i.e., in the first sum only $d_t(t) = 1$, in the second $d_{t-1}(t-1) = 1$ and so on. We also include the subscript i in the dummy variable in the fourth line to make explicit that the sequence of dummies depends on item i.

Now, consider the term $\tilde{a}_{i,t} - \tilde{a}_{i,\tau_{i,t}}$. Under Assumption 3 we have:

$$\begin{aligned} \tilde{a}_{i,t} - \tilde{a}_{i,\tau_{i,t}} =& (\eta + \cdots + \eta) + (\varepsilon_{i,t} + \cdots + \varepsilon_{i,t-\delta_{i,t}+1}) \\ =& \eta\delta_{i,t} + u_{i,t}, \end{aligned}$$

where $\delta_{i,t}$ is the time interval between t and $\tau_{i,t}$, and $u_{i,t} = \varepsilon_{i,t} + \cdots + \varepsilon_{i,t-\delta_{i,t}+1}$ is a moving average MA($\delta_{i,t} - 1$) process. Therefore, $u_{i,t} \sim \mathbb{N}(0, \delta_{i,t}\sigma^2)$.

Then, putting all parts together we obtain:

$$r^*_{i,t} = p_{i,\tau_{i,t}} - p^*_{i,t} = c - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T}\gamma_j d_{i,t}(j) - u_{i,t}.$$

C Additional estimation results

Table 6: Detailed results of the probit model, baseline specification

Baseline specification					
Log-likelihoood: -3.0634e+06				Number of obs:	$2,243,642$
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.62	0.01	71.59	0.61	0.64
$(c-S)/\sigma$	-0.92	0.01	-106.19	-0.93	-0.90
$\sqrt{\delta_{i,t}}$	-0.39	1.63	-0.24	-3.58	2.79
$\mathcal{Y}_t$	0.60	0.08	7.76	0.45	0.75
p_t	9.57	0.05	194.12	9.48	9.67
e_t	0.11	0.01	9.12	0.09	0.13
ζ	0.06	0.01	-	0.04	0.07
σ	0.10	0.01	-	0.08	0.11

Table 7: Detailed results of probit models, specifications for robustness

Specification with sample 1997-2000

Log-likelihoood: -8.9567e+05 Number of obs: 666,948

Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.53	0.02	22.52	0.48	0.58
$(c-S)/\sigma$	-0.76	0.02	-32.68	-0.81	-0.72
$\sqrt{\delta_{i,t}}$	-0.02	0.02	-1.19	-0.06	0.02
$\mathcal{Y}_t$	0.15	0.16	0.91	-0.17	0.46
p_t	4.93	0.15	33.67	4.64	5.21
e_t	0.62	0.02	25.53	0.57	0.67
ζ	0.03	0.03	-	-0.03	0.09
σ	0.19	0.03	-	0.13	0.26

Specification with sample 2001-2004

Log-likelihoood: -1.6960e+06 Number of obs: 1,277,736

Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.54	0.01	42.45	0.51	0.56
$(c-S)/\sigma$	-0.93	0.01	-75.12	-0.96	-0.91
$\sqrt{\delta_{i,t}}$	-0.05	0.01	-4.25	-0.08	-0.03
$\mathcal{Y}_t$	1.29	0.09	13.66	1.10	1.47
p_t	11.07	0.05	204.29	10.97	11.18
e_t	-0.40	0.02	-26.35	-0.43	-0.37
ζ	0.10	0.01	-	0.09	0.12
σ	0.08	0.01	-	0.07	0.09

Specification without exchange rate

Log-likelihoood: -3.0698e+06 Number of obs: 2,243,642

Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.60	0.01	107.17	0.58	0.61
$(c-S)/\sigma$	-0.94	0.01	-171.69	-0.95	-0.93
$\sqrt{\delta_{i,t}}$	-6.10	0.08	-74.61	-6.26	-5.94
$\mathcal{Y}_t$	2.93	0.07	39.51	2.78	3.07
p_t	3.43	0.04	93.05	3.36	3.50
ζ	0.46	0.01	-	0.45	0.48
σ	0.16	0.01	-	0.14	0.18

Specification with idiosyncratic white noise

Log-likelihoood: -2.2784e+06 Number of obs: 2,243,642

Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.82	0.01	82.19	0.80	0.84
$(c-S)/\sigma$	-1.19	0.01	-118.74	-1.21	-1.17
$\mathcal{Y}_t$	0.90	0.11	7.99	0.68	1.12
p_t	12.72	0.07	175.79	12.58	12.87
e_t	0.05	0.02	2.72	0.01	0.08
ζ	0.07	0.01	-	0.05	0.08
σ	0.07	0.01	-	0.06	0.09

Table 8: Detailed results of probit models, specifications for robustness (continuation)

Specification without treating promotions					
Log-likelihoood: -3.0779e+06				Number of obs:	$2,243,642$
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.59	0.01	67.35	0.57	0.60
$(c-S)/\sigma$	-0.88	0.01	-101.90	-0.90	-0.86
$\sqrt{\delta_{i,t}}$	-0.04	0.31	-0.13	-0.64	0.56
$\mathcal{Y}_t$	0.53	0.08	6.75	0.38	0.69
p_t	8.90	0.05	176.56	8.80	9.00
e_t	0.02	0.01	1.80	0.00	0.05
ζ	0.06	0.01	-	0.04	0.07
σ	0.11	0.01	-	0.09	0.12

Specification without itens exclusions					
Log-likelihoood: -3.8638e+06				Number of obs:	$2,851,318$
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.63	0.01	76.81	0.61	0.64
$(c-S)/\sigma$	-0.93	0.01	-113.99	-0.94	-0.91
$\sqrt{\delta_{i,t}}$	-0.35	0.06	-5.90	-0.47	-0.23
$\mathcal{Y}_t$	0.44	0.07	6.79	0.31	0.57
p_t	9.79	0.04	224.11	9.70	9.88
e_t	0.03	0.01	3.07	0.01	0.05
ζ	0.04	0.01	-	0.03	0.05
σ	0.10	0.01	-	0.09	0.11

D Products excluded because of potential measurement errors

In this appendix we document the rationale for excluding some products from our database due to problematic price quotes. We also list those products excluded in the estimation of the baseline specification. By problematic price quotes we mean those prices in which there is evidence that, for technical reasons (the method used to measure and/or to collect prices), the recorded values not necessarily refer to the same individual product along the time, i.e., we are not sure that the prices recorded in the database are effectively micro prices in the sense we need in our estimations. In all cases, by maintaining these items, we could be computing a price change when it actually did not happen. These problematic price quotes arise from a variety of measurement problems, which we organize in five broad categories.

D.1 Micro prices hard to collect:

There are products whose prices, for several reasons, are hard to collect and, consequently, there are doubts if they are actually micro prices. In some cases the difficulty arises from the fact that is hard to find the same product, with the same characteristics, along the time. In this group, "used cars" is a good example. In other cases, the method of price collection changes among itens and/or over time. Here "fish" is an example. Sometimes the price of fish is collected per pound, sometimes by package and also by piece. When calculating the CPI these changes are handled, but we cannot identify them in the dataset.

Products excluded: Shrimp (cod 112507), Corvina fish (cod 112515), Fresh sardine (cod 112529), European hake (cod 112536), Salmon (cod 112562), Videotape (cod 530753), Compact disc (cod 530755), Ticket for concerts (cod 530905), Ticket for theater (cod 530907), Ticket for soccer and other sports events (cod 531303), Airfare (cod 531505), Used cars (cod 620103), Telephone card (cod 720101)

D.2 Index number

Prices of some products are not individually registered in the dataset, but only in the form of an index. For residential rent, for example, the database does not record the price of the rent for each apartment individually. Instead, the price collectors visit many apartments with the same characteristics (e.g. two-bedroom apartments) and calculate the average price. This "rental index for two-bedroom apartments" is what is recorded in the dataset. Similar procedure is used for other products, such as condominium fees and housemaids.

Products excluded: Residential rents (cod 210101), Condominium fees (cod 210103), Daily housemaid (cod 280101), Monthly housemaid (cod 280103)

D.3 Prices of a basket of goods

Because of their very distinct characteristics, for some goods the procedure involves collecting prices for a basket of goods with similar characteristics and recording only the average price. Typical goods in this category are the items of clothing. For instance, it is very hard to collect prices for the same shirt along the time. In this case, prices of a basket of shirts with very similar characteristics (e.g. men's shirts) are collected and the average price is recorded in the dataset.

Products excluded: Residence furniture (cod 230111), Mattresses (cod 230703), Blankets (cod 240101), Sheets and pillowcase (cod 240103), Towels (cod 240105), Curtains and blinds (cod 260101), Rug and carpet (cod 260115), Drinking glass (cod 260305), Pots (cod 260309), Men's shorts (cod 310101), Men's pants (cod 310105), Men's shirts (cod 310111), Men's underwear (cod 310117), Women's shirts (cod 310307), Women's pants (cod 310309), Women's underwear (cod 310313), Dresses and skirts (cod 310331), Children's shorts (cod 310503), Children's shirts (cod 310505), Children's pants (cod 310507), Men's shoes (cod 320105), Men's tennis shoes (cod 320107), Women's sandal (cod 320303), Women's tennis shoes (cod 320305), Women's shoes (cod 320307), Children's tennis shoes (cod 320511), Eyeglasses (cod 420303)

D.4 Prices collected annually or semiannually

Educational fees and tuition are known to change on an annual or semiannual basis and, therefore, price collection is generally made in the same frequency.

Products excluded: Elementary education (cod 510101), High school education (cod 510103), Kindergarten (cod 510105), College education (cod 510107), Day-care (cod 510153), Graduate education (cod 510317)

D.5 Administered prices

This group of prices not necessarily suffer from problematic price quotes, but we decided to exclude them from our estimations because they are administered by contract or monitored, i.e., they are relatively insensitive to supply and demand conditions or are in someway regulated by a public agency.

Products excluded: Water and sewer service charge (cod 210305), Residential electricity rates (cod 220101), Cylinder gas (cod 220103), Residential gas (cod 220105), Home phone service (cod 220109), Analgesic and antipyretic (cod 420111), Anti-infective drugs (cod 420112), Antiinflammatory (cod 420114), Antitussive (cod 420115), Anti-allergy drugs (cod 420116), Blood pressure medication (cod 420117), Antidepressants (cod 420118), Contraceptive pills (cod 420123), Subway fare (cod 610103), Bus fare (cod 610105), Intercity bus fare (cod 610303), Gasoline (cod 620503), Toll tariffs (cod 620909), Lottery game (cod 720305)