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Informality in developing economies: regulation and fiscal policies

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Abstract

This paper proposes a unified theoretical framework where formal and informal firms coexist and face the same type of product and labor market imperfections: they have monopoly power in the goods market, they are subject to matching frictions in the labor market, and wages are determined through bargaining between large firms and their workers. Our model matches the main stylized facts on informality for developing countries and appears to be a good candidate for policy analysis. In this framework, we study the impact on informality, wages and unemployment of two types of regulatory policies that may be used to reduce informality, labor and product market regulation, and two types of fiscal policies, labor taxes and formality enforcement. We find that lessening regulation decreases informality and unemployment simultaneously, indicating that there is not necessarily a tradeoff between informality and unemployment. The tradeoff appears when fiscal policies are used, though. Additionally, a reduction in labor market regulation is the only informality-reducing policy considered that diminishes wage inequality.

Keywords Informality; Product and Labor Market Imperfections; Search and Matching

JEL Classification E24; E26; J60; L16; O1

1 Introduction[1]

Informal activities are pervasive in both developed and developing economies. According to the estimates of Schneider and Enste (2000), the size of the shadow economy as a percentage of GDP ranges from 25 to 60% in Latin America, from 13 to 50% in Asia, and is around 15% among OECD countries, reaching 30% in some European countries. Informal firms differ from formal ones in a number of measurable characteristics, and there is a growing literature trying to understand the causes of informality and its differences from formal businesses. Although widespread, informality is associated with a number of undesirable features. Not only do informal firms tend to be less productive and to pay lower wages, but their very existence holds in check the rule of law and the integrity of public institutions. Policies aimed at reducing informality, however, may have some undesirable effects. It is generally argued that unemployment and informality are the two faces of the same coin and that policy makers cannot contend with the latter without harming the former. In addition, such policies may also induce awkward effects on wage inequality. It follows that informality-reducing policies may come at a high political cost which is not likely not be sustainable by legal authorities.

The aim of our paper is to investigate the overall impact of policies intended to reduce informality. In particular, we seek to detect whether a tradeoff between unemployment and informality exists and what the relevant policies from this standpoint are, as well as in other dimensions such as wage inequality.

The novelty of our approach is to propose a unified theoretical framework that matches the main stylized facts on informality for developing countries and appears consequently to be a good candidate for policy analysis. Our modeling strategy innovates by combining a number of features as being shared simultaneously by both formal and informal sectors. In sum, in our model: (i) there are frictions in both formal and informal labor markets; (ii) job seekers are identical and can find jobs in both sectors, that is, the labor market is not segmented; (iii) the number and size of firms in each sector is endogenous, which renders the degree of competition

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in each sector endogenous; (iv) wages are set through bargaining between large firms and their workers. As will be discussed below, this paper is the first to simultaneously incorporate all these features and allows us to shed new light on the links between unemployment, wages and informality.

We simulate the model numerically to show how changes in policy variables affect informality, unemployment and wages. In order to illustrate the functioning of the model with realistic quantities, we use the Brazilian economy to guide our parametrization, since it represents well the type of economy we model.

In this framework, we study the effects of regulatory and fiscal policies in particular on informality, unemployment and wages. We find that less regulation reduces both informality and unemployment. The effects of labor market regulation (LMR) are much stronger compared to those of product market regulation (PMR). A fall in LMR that reduces informality by 1% would decrease unemployment by 1.49%, whereas the unemployment reduction would be only 0.15%, for a fall in PMR that generates the same reduction in informality. Regarding wages, lessening LMR decreases wages, but it also lowers the wage differential between formal and informal workers. On the other hand, reducing PMR increases wages along with an increase in wage inequality. The negative impact of LMR on wage inequality is a non-negligible argument in favor of using LMR to diminish informality in countries with high levels of inequality, such as Latin American countries. Alternatively, this suggests that it is possible to simultaneously lessen product and labor market regulations to achieve a reduction in unemployment and informality without increasing wage inequality.

With respect to fiscal policies, lowering taxes and increasing the detection rate of informality do reduce informality, but these policies also raise unemployment. Nevertheless, tax reduction has a smaller impact on unemployment compared to an increase in formality enforcement. A reduction in taxes that decreases informality by 1% would increase unemployment by 0.19%, while unemployment would increase by 0.35% for a rise in formality enforcement that also reduces informality by 1%. The impact of lower taxes on wages is similar to that of less PMR: higher wages and higher inequality. As for formality enforcement, it decreases wages and increases inequality at the same time.

All in all, regulation dominates fiscal policies as measures to boost formality, for unemployment decreases with lessened regulation whereas it rises when fiscal policies are used. Increasing informality detection is the least preferable policy, since all its side-effects are negative: it increases unemployment, reduces wages and increases inequality.

Our paper is related to at least two strands of the literature.

First, following Blanchard and Giavazzi (2003), a number of recent papers have studied the impact of PMR and LMR on unemployment in economies with frictions in the labor market (e.g. Delacroix, 2006, Ebell and Haefke, 2009, Felbermayr and Prat, 2011). These studies, however, do not consider the existence of an informal sector. Given the significant share of informality in most economies, it is important to understand how its presence may affect the impacts of changes in regulatory policies. For instance, the relative size of the informal sector should be responsive to changes in the costs involved in creating a new business, since many such costs are avoided by firms entering the informal sector. Indeed, Figure 1 below illustrates that, among Latin American countries, the informal sector tends to be larger in countries where barriers to entry are stricter. Many developing countries tend to have relatively higher barriers to entry of new businesses (see Djankov, La Porta, Lopez de Silanes and Shleifer, 2002), which partly explain why informality is more pervasive among those countries.

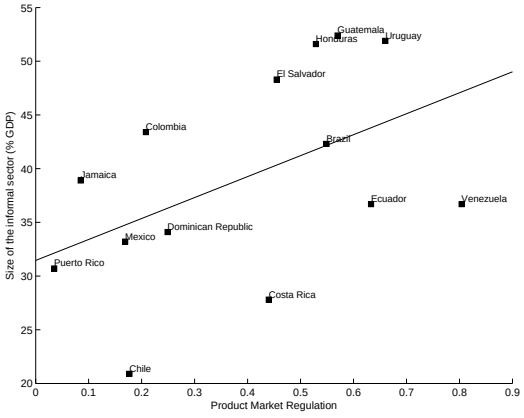

Figure 1: Informality and PMR in developing countries

Second, there is a recent and growing literature on unemployment and informality in developing countries. Most of the studies in this literature, such as Fugazza and Jacques (2003), Satchi and Temple (2009) and Zenou (2008), take the formal sector as a disadvantaged or residual sector. Based on the Latin American experience, Maloney (2004) and Fiess, Fugazza and Maloney (2010) claim that the informal sector should be viewed as an unregulated micro-entrepreneurial sector instead. There is some controversy on this issue, particularly when considering rural and urban labor markets. It is fair to say, though, that formal and informal labor markets are integrated at least in urban areas, which is the focus we choose for our paper. In terms of the behavior of the unemployed, (urban) job seekers in developing countries are

likely to look simultaneously for formal and informal jobs, either because they cannot afford to do otherwise, or because there is less social stigma attached to taking a job in the informal sector.

Various alternative assumptions have been considered in the literature. Zenou (2008) studies the impact of labor market policies on informality, and he models the formal sector as subject to labor market frictions and presenting unemployment, while the informal one is taken as being competitive. Although it is generally acknowledged that one of the advantages of the informal sector lies in the fact that finding a job is easier, there is at best no compelling evidence that the informal labor market is a fully competitive one. In any event, this particular case can easily be embedded in a more general model incorporating matching frictions in both formal and informal labor markets, such as the one developed in this paper. Moreover, the numerical simulation of our model does generate endogenously a more competitive informal sector, which is consistent with stylized facts.

Kolm and Larsen (2006) and Ulyssea (2009) assume, as we do, that unemployed workers seek both formal and informal jobs, and that both types of job are subject to matching frictions. The former analyzes the effects of higher punishment and audit rates on labor market performance, while the latter focuses on endogenous differences in productivity between the two sectors. We depart from these considerations to focus on differences in firm size across sectors, which renders their relative degree of competition endogenous.

Alternatively, Boeri and Garibaldi (2006) and Albrecht, Navarro and Vroman (2009) are interested in explaining the sorting of workers across sectors, and they assume that workers differ in their productivity[2]. We are aware of the empirical evidence suggesting differences in workers skills and firms' productivity in formal and informal sectors, but the aim of our paper is not to explain these features. We focus, instead, on explaining differences in the size of firms and the degree of competition across sectors, as well as the impact of changes in regulations and fiscal policies on unemployment, on the relative size of informal and formal sectors, and their relative wages.

An assumption common to all these previous models is that each firm is allowed to hire only one worker. We depart from this assumption and let firms hire as many workers as they desire. El Badaoui, Strobl and Walsh (2010) and Meghir, Narita and Robin (2011) develop, to our knowledge, the only alternative model in this literature in which firm size is also an

[2]See also Basu, Chau and Kanbur (2011) who build a search model where firms choose to become formal or informal while workers differ in ability.

endogenous variable. Based on Burdett and Mortensen (1998), they build models with on-the-job search and wage posting (instead of wage bargaining) where firms' choice of wages determines their size. In our model, however, firms choose their size directly and wages are a result of a bargaining process between large firms and their workers. Hence, the number and size of firms are endogenously determined, which renders the degree of competition in each sector endogenous. Our model generates a more competitive informal sector, which is in line with empirical evidence. Additionally, we find that the formal sector draws closer to the informal one in terms of degree of competition for all policies considered that reduce informality.

The paper is organized as follows. Section 2 presents some key stylized facts about informality in Brazil. The theoretical model is described in Section 3. Section 4 presents a quantitative assessment of the model, as well as policy exercises. In addition, in section 5, we run a number of counterfactual experiments meant to evaluate the importance of the various features at play in our model and to check our results under alternative hypotheses. Section 6 concludes.

2 Some evidence for a large developing country

This section presents a series of important stylized facts that are relevant for Latin American countries. Note that while we do not have a particular country in mind, these facts will serve to parameterize our benchmark economy with realistic values. Our main source of data comes from a large Latin American country, namely Brazil, that corresponds well to the type of economy we want to represent. In practice, we use data from the Monthly Employment Survey (Pesquisa Mensal de Emprego, or PME) conducted by the Brazilian Institute of Geography and Statistics (IBGE) for the greater metropolitan regions of São Paulo, Rio de Janeiro, Belo Horizonte, Porto Alegre, Salvador and Recife. PME collects information on employment and earnings, as well as on other observable characteristics such as the worker's years of schooling, age, gender, state of residence, sector of activity and occupation.

In Brazil, all workers formally employed in the private sector are required to have a signed working card (*carteira de trabalho*). Thus, by observing whether the individual has a valid working card signed by his or her employer we are able to sort formal and informal workers.[3]

[3]Any Brazilian resident may request a working card at no cost and hardly any bureaucracy. When the person is formally employed, it is mandatory that the employer signs his or her working card. Notice that we also need to know whether the individual works in the public sector, since the working card is not signed for public servants. This question was included in the PME questionnaire from 2002 on. Hence we choose to use data starting in 2002. For data prior to 2002 it was not possible to sort informal workers from those working in the public sector in the PME survey.

Among the self employed, it is also possible to distinguish those who pay social contributions from those who do not. We then define informal workers as those informally employed in the private sector and the self employed who do not pay social contributions. As shown in Figure 2, from 2003 to 2010 the share of the informal sector declined from around 40 to 35% in Brazil, while the unemployment rate also decreased from around 12 to 7%.

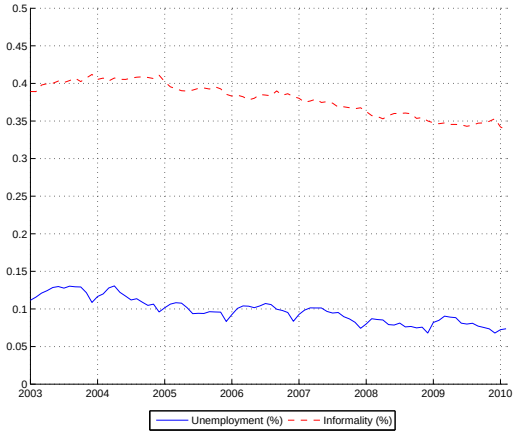

Figure 2: Informality and unemployment in Brazil

PME interviews the same individual at different moments in time for a period of 16 months. For each individual, four interviews are conducted over the first four months, then there is an interval of eight months, and once again the same individual is interviewed for four consecutive months. Thus the information is gathered in months t, t+1, t+2, t+3, t+12, t+13, t+14 and t+15. We use the fourth (t+3) and eighth (t+15) interviews of each individual to compute the transition frequencies across employment states. Table 1 below presents the transitions across unemployment, formality and informality, where each line displays the state of origin and the column the destination state. The table reveals some interesting patterns. An unemployed worker has virtually equal probability of being in any one of the three states one year later. Formal workers have a probability of 87.4% of remaining formal, while only 71.7% of informal workers remain informal after one year. Finally, informal workers become formal with a probability of 23.1%, whereas the reverse is true with a frequency of only 9.3%. This empirical evidence suggests that the Brazilian labor market is not segmented for job seekers, and that the informal sector features a substantial degree of search frictions.

We have also computed the same transition matrix using two alternative sub-samples.[4] In

[4]The results are available upon request.

State T \ State T+1	Unemployed	Employed: Registered	Employed: Unregistered
Unemployed	35,8%	32,1%	32,1%
Employed: Registered	3,4%	87,4%	9,3%
Employed: Unregistered	5,3%	23,1%	71,7%
Total	6,6%	62,2%	31,2%

Total Observations (Different Individuals)	246 574
Average Observations per Month	3 378
Overall Period	Jan 2003-Feb 2010
Estimation Sample	Jan 2004-Feb 2010
Source	Monthly Labor Survey (PME) - IBGE

Table 1: Transition probabilities

first one we restrict the sample to workers 23 and 65 years old, which corresponds to 155,002 observations. In the second subsample we consider only low-skill workers (those with lower than high-school education), which amounts to 124,569 observations. In all cases, we get broadly the same picture, with marginal differences. In particular, there are more marked differences in the probabilities for an unemployed individual to find a formal or an informal job using these subsamples. In the first subsample, among the unemployed at time t, 34.5% stay unemployed, 31.1% find a job in the formal sector, while 34.4% become an informal worker at $t+1$. In the second subsample, the same percentages are 34.9%, 29.4 and 35.7% respectively.

The formal sector wages are 45 to 55% higher than wages in the informal sector. We know, however, that formal and informal sector workers differ in a number of characteristics that affect wages. We therefore estimate the wage premium in the formal sector after controlling for observable individual characteristics available in the data.[5] Figure 3 shows the path of the controlled wage gap from 2007 to 2010 and its confidence interval of 95%. Formal sector wage premium ranged from 23 to 30%, which is indeed much smaller than the observed one but still considerable. Note also that the formal sector wage premium decreased by approximately 20% over the period.[6]

With respect to firm size, PME asks the employee to place the size of the firm where he or she works in one of three categories: the first one corresponds to firms with 2 to 5 employees,

[5]We run Mincer regressions in cross section for each month, where wages are explained by a dummy for informal workers, a dummy for each city, and the following worker characteristics: age, age square, education, education square, and position in the household.

[6]To be precise, within the 95% confidence interval, the decrease in wage premium ranged between 7% and 31%.

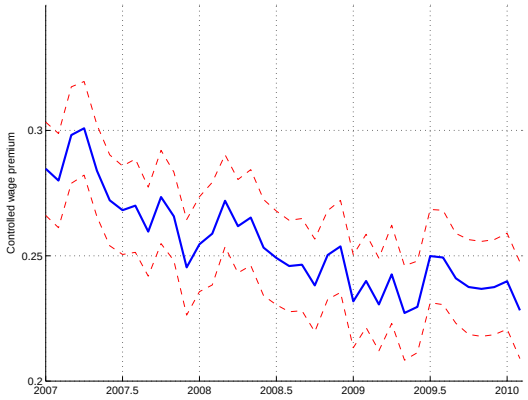

Figure 3: Wage Premium

the second one with 6 to 10 employees, and the third with 11 employees or more. We create a size index by assigning a number from 1 to 3 to each of these categories. Figure 4 presents the averages of this size index for the informal and formal sectors. Formal sector firms are clearly larger than informal ones.

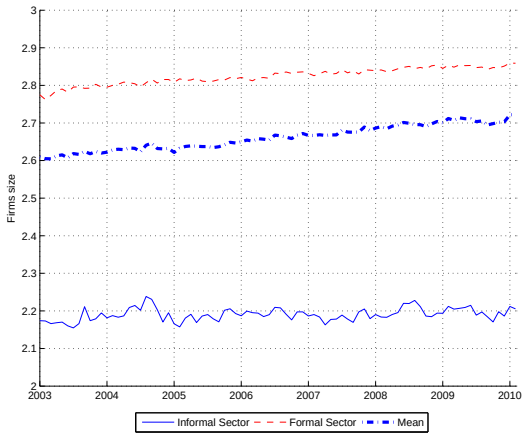

Figure 4: Firm size in formal and informal sectors

3 The model

We build a multi-sector model economy capturing the main features of a developing economy as described in the previous section. We assume that there are two sectors in the economy, a formal and an informal one, each producing one of the two consumption goods available. In each sector there is an endogenous number of firms competing in quantities and producing a

homogeneous good. The two types of goods, formal and informal, are substitutes. The labor market is integrated, but there are matching frictions in both sectors. We proceed as follows. First, we present the goods and the labor markets. Second, we detail firms' optimal behavior and the wage bargaining process. Lastly, we define the equilibrium.

3.1 The goods market

We assume that there is some substitutability between the goods produced by the informal and formal sectors, but firms within each sector produce a homogeneous good. The idea we want to capture can be illustrated by the soft drink market in Brazil. The formal sector produces the usual brands of soft-drinks, such as Coca-Cola and Pepsi. However, a significant share of the market, particularly in poorer regions, is captured by soft drinks locally produced mostly by informal firms. They are generically called '*tubainas*'. Our assumption is that the 'colas' and '*tubainas*' are homogeneous among themselves, but differentiated from each other.

Households are both consumers and workers, and derive utility from consuming goods from both formal and informal sectors. Their preferences are represented by the following CES utility function:

$$\mathcal{U} = \left[\alpha_F^{\frac{1}{\sigma}} C_F^{\frac{\sigma-1}{\sigma}} + \alpha_I^{\frac{1}{\sigma}} C_I^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where C_j denotes a household's consumption of the good produced in sector j, for $j = \{F, I\}$ where F and I refer to the formal and the informal sector respectively. σ stands for the elasticity of substitution between the two goods and α_j is a preference parameter. Assuming, for the sake of simplicity, that $\alpha_F = \alpha_I \equiv \alpha$, the optimal consumption pattern of a household n with real income R_n is given by:[7]

$$C_{jn} = \alpha R_n \left(\frac{p_j}{P}\right)^{-\sigma} \tag{2}$$

where p_j is the price of sector j good, P is the composite price index, $P = \left(\alpha \sum_{j=I,F} p_j^{1-\sigma}\right)^{1/(1-\sigma)}$. There is a continuum of identical consumers in the interval $[0, 1]$, hence aggregate consumption is:

$$C_j = \alpha R \left(\frac{p_j}{P}\right)^{-\sigma}, \tag{3}$$

where $R \equiv \int_0^1 R_n \mathrm{d}n$ denotes aggregate income.

There are N_j identical firms in sector j engaging in Cournot competition. We restrict our attention to symmetric equilibria within each sector, according to which all firms produce

[7] See e.g. Delacroix (2006) for a similar expression.

an equal share of the total demand for the sector. We then have that $C_j = N_j y_j$, where y_j denotes the production of a firm in sector j. Normalizing the aggregate price index, P, to unity, symmetric Cournot competition implies an inverse demand function that satisfies:

$$p_j = \left(\frac{N_j y_j}{\alpha R} \right)^{-\frac{1}{\sigma}}. \tag{4}$$

In this way, as a result of Cournot competition among firms, the elasticity of demand faced by a firm in sector j, σ_j, is positively related to the number of firms operating in that sector as follows:

$$\sigma_j \equiv \sigma_j(N_j) = \sigma N_j. \tag{5}$$

Thus, in this simple framework, the number of firms, N_j, determines the level of competition for a firm in sector j.[8]

Labor is the only input in production. Recent research on informality has highlighted the fact that informal firms are less productive than formal ones (see, for instance, Lemieux, Fortin, and Fréchette, 1996, La Porta and Shleifer 2008, Taymaz, 2009). In particular, in a careful empirical study on informality La Porta and Shleifer (2008) find that on average the productivity differential between small informal (unregistered) firms and small formal (registered) firms is in the range of 71 to 314%. We allow for productivity differences across sectors by defining the production function as:

$$y_j = A_j h_j \tag{6}$$

where A_j stands for a (sector-specific) productivity parameter and h_j is the firm's size, that is, the number of workers employed.

The size of firms will be endogenously determined in the model. We will see that, in equilibrium, firms in the informal sector are smaller, which corroborates the facts described in section 2 and the numerous results in the literature (see e.g. Rauch, 1991 or Tybout, 2000).

3.2 The labor market: matching frictions

We assume that both formal and informal sectors are subject to matching frictions and that they share the same pool of unemployed workers.[9] These two assumptions depart from other

[8] An alternative modeling strategy with imperfect competition in the goods market can be found in e.g. Blanchard and Giavazzi (2003), Ebell and Haefke (2009), or Felbermayr and Prat (2011). In all three cases, they assume that the elasticity of demand is positively related to the number of firms, whereas in our case, under Cournot competition, the relation between the elasticity and the number of firms is endogenous.

[9] For comparison purposes, we relax this assumption in section 5.3 and we explore the alternative case of full segmentation of the labor market between formal and informal jobs.

recent works that incorporate search-matching frictions to study informality, which either assume a segmented labor market or take the informal sector as a residual perfectly competitive sector. Rather, we take the view that, while the informal sector fails to comply with taxes and government regulations, it is otherwise similar to the formal one. This approach is consistent with e.g. Maloney (1999, 2004).

We are aware of empirical evidence that indicates that informal workers have on average lower educational attainment levels (see, among others, Gong and Van Soest, 2002, Gong, Van Soest and Villagomez, 2004, and Maloney, 2004). Informality would therefore concern low skill workers to a greater extent than high skill workers. We choose to restrict ourselves to identical workers, focusing on low skill workers only.[10] This hypothesis is not necessarily too restrictive. La Porta and Shleifer (2008) present evidence that, while top managers in informal firms are indeed less educated, other employees have a comparable level of education in informal and in small formal firms.

In each sector, vacant jobs and unemployed workers are brought together in pairs by a matching function M_j. It maps the number of matches in sector j to the total number of job seekers and vacancies in that sector, $M_j \equiv M_j(u, v_j)$ where u and v_j correspond respectively to the mass of job seekers, which is the unemployment rate in the economy [11], and to the mass of vacancies in the sector. The function M_j features standard properties: it is twice continuously differentiable, increasing and concave in both of its arguments, it is linearly homogeneous, satisfies the Inada conditions and the boundary conditions $M_j(0, v_j) = M_j(u, 0) = 0$ for $u, v_j \geq 0$.

On average, a firm contacts a worker at rate M_j/v_j while a job seeker meets a sector j firm at rate M_j/u. Let $\theta_j \equiv v_j/u$ be the labor market tightness. Linear homogeneity of the matching function allows us to write those contact rates as $M_j/v_j \equiv m_j(\theta_j)$, with $m_j'(\theta_j) < 0$, and $M_j/u = \theta_j m_j(\theta_j)$, which is an increasing function of θ_j. Thus vacancies are filled at rate $m_j(\theta_j)$ in sector j while workers exit unemployment at rate $\sum_{j=F,I} \theta_j m_j(\theta_j)$.

We allow the matching function to be different across sectors to be able to capture their particularities. For instance, firms may rely on different methods to recruit their workers in the two sectors, so that the efficiency of the two matching processes may differ somewhat.

[10] This is of course a simplification compared to the papers studying the sorting mechanism between the two sectors (e.g. Boeri and Garibaldi, 2006, Albrecht et al., 2009), but our focus is not on explaining sorting patterns across sectors.

[11] We assume job seekers are 'truly' unemployed. Alternatively, we could assume that they are in fact self-employed, and that the self-employed search for jobs.

It is also often argued that, compared to the formal sector, the informal sector is closer to a competitive market where it takes less time to match. In that case, the matching process would be more efficient in the informal sector, resulting in more matches for the same level of inputs in the matching function. More specifically, in our numerical simulations we use a Cobb-Douglas function for the matching function, $M_j = \kappa_j u^{\eta} v_j^{1-\eta}$, with $\kappa_I \geq \kappa_F$ to capture the easier matching in the informal sector.

Note that the assumption of search frictions in both sectors encompasses the particular case of perfect competition in the informal labor market considered in several papers, which would correspond to the limit case where $\kappa_I \to \infty$ in our framework. Furthermore, if it took no time to locate a formal job ($\theta_F m_F(\theta_F) \to \infty$), all workers would be employed in the formal sector. Thus, matching frictions are an important reason for the existence of an informal sector, and the underlying assumption for the existence of the two sectors is that the formal sector is 'sufficiently' frictional.

Matches are dissolved at rate d_j, due either to an exogenous separation rate s_j between firms and workers or to the exit of firms from the market, which occurs with probability δ_j. Hence, the sector-specific destruction rate is:

$$d_j = \delta_j + s_j(1 - \delta_j) \tag{7}$$

Workers can be either employed or unemployed so that:

$$\sum_{j=F,I} L_j = 1 - u \tag{8}$$

where $L_j = N_j h_j$ denotes employment in sector j, and u stands for unemployment. In steady-state equilibrium, the mass of unemployed workers that find a job in a sector has to equal the mass of workers that lose a job in that sector, that is:

$$d_j L_j = \theta_j m_j(\theta_j) u \tag{9}$$

Equation (9) states that when a fraction d_j of the jobs in sector j are destroyed, they are compensated by an inflow $\theta_j m_j(\theta_j) u$ of job seekers who are recruited in sector j.

3.3 Firms problem

Following a growing body of the literature, we depart from the basic matching model by assuming firms can hire more than one worker. This implies that firm size becomes an endogenous

variable which responds to changes in the firm's expected profits. This feature of the model allows us to analyze the determinants of the relative size of firms in the two sectors. In particular, firm size depends on the elasticity of substitution σ_j, as we will show below, due to the assumption of imperfect competition in the goods market.

Firms choose the number of vacancies and their size so as to maximize expected profits, which can be written as:

$$V_j(h_j) = \max_{v_j, h'_j} \frac{1}{1+r} \left[p_j\left(y_j\left(h_j\right)\right) y_j\left(h_j\right) - w_j\left(h_j\right) h_j \left(1+\tau_j\right) - \gamma_j v_j + \left(1-\delta_j\right) V_j\left(h'_j\right) \right], \quad (10)$$

where r is the interest rate, γ_j is the cost of a vacancy, τ_j represent taxes on labor costs, and $w_j\left(h_j\right)$ is the wage function resulting from a bargaining process, which will be derived in the next section. h_j and h'_j represent the number of workers in current and next periods, respectively. The production function $y_j\left(h_j\right)$ is in equation (6) and the inverse demand function $p_j(y_j)$ is derived from demand in equation (3).[12]

Notice that, in this setting, firms do not take prices as given in the final goods market, as shown by the inverse demand function, and they enjoy some bargaining power in the labor market, which is captured by the wage function.

The number of workers in the next period, h'_j, is determined by the following transition function:

$$h'_j = m_j(\theta_j) v_j + (1 - s_j) h_j, \quad (11)$$

that is, the next period's employment is equal to the number of matches for the vacancies posted plus the number of current workers that remain employed. Thus, firms advertise as many vacancies as necessary in order to hire, in expected value, the desired number of workers next period, taking into account the cost of a vacancy γ_j and the constraints on labor market flows given by the transition function (11).

The optimal number of vacancies posted is such that the marginal contribution of a worker to the firm's expected profit is equal to the expected search cost, that is:

$$(1-\delta_j) \frac{\partial V_j(h'_j)}{\partial h'_j} = \frac{\gamma_j}{m_j(\theta_j)}. \quad (12)$$

From the profit function (10), the marginal contribution of a worker to the firm's profit,

[12] More precisely, price is a function of total production in the sector, $p_j \equiv p_j(y_j + (N_j - 1)y_{-j})$. With some abuse of notation, we express price as a function of one firm's production, implicitly taking the production of all other firms as given. In symmetric equilibrium all firms produce the same quantity and $y_j = y_{-j}$.}

denoted the envelope condition, can be written as:

$$\frac{\partial V_j\left(h_j\right)}{\partial h_j} = \frac{1}{1+r}\left[\begin{array}{c} \frac{\sigma_j-1}{\sigma_j}p_jA_j - \left(w_j(h_j) + \frac{\partial w_j(h_j)}{\partial h_j}h_j\right)(1+\tau_j) + \\ +\left(1-\delta_j\right)\left(1-s_j\right)\frac{\partial V_j\left(h_j'\right)}{\partial h_j'} \end{array}\right], \tag{13}$$

where we have used the fact that $\frac{\partial p_j(y_j)}{\partial y_j}\frac{\partial y_j}{\partial h_j}y_j + p_j\frac{\partial y_j}{\partial h_j} = \frac{\sigma_j-1}{\sigma_j}p_jA_j$ and that $\frac{\partial h_j'}{\partial h_j} = 1-s_j$. The term $\frac{\sigma_j-1}{\sigma_j}p_jA_j$ corresponds to marginal revenue from which the marginal costs $\left(w_j(h_j) + \frac{\partial w_j(h_j)}{\partial h_j}h_j\right)\times (1+\tau_j)$ of expanding the labor force to h_j should be subtracted. Marginal costs differ from wage since firms take into account the effect of an additional worker on the wages of previously employed workers.

Substituting the envelope condition (13) into the optimal vacancies condition (12), we get that, in steady state:

$$p_j(h_j) = \frac{1}{A_j}\left(\frac{\sigma_j}{\sigma_j-1}\right)\left[\left(w_j(h_j) + \frac{\partial w_j(h_j)}{\partial h_j}h_j\right)(1+\tau_j) + \frac{\gamma_j(r+d_j)}{m_j(\theta_j)(1-\delta_j)}\right]. \tag{14}$$

Equation (14) defines firms' pricing behavior in steady state, and can be interpreted as a mark-up equation over total labor costs, inclusive of wages and search costs. Firms enjoy some market power on the goods market, but also on the labor market, due to the existence of search frictions. In the absence of frictions, price p_j would simply be equal to $\frac{1}{A_j}\left(\frac{\sigma_j}{\sigma_j-1}\right)w_j$. Here, though, the marginal cost of labor also takes into account the existence of recruitment costs and the impact of an additional worker on the wages of the infra-marginal workers, $\frac{\partial w_j(h_j)}{\partial h_j}h_j$. The latter term is negative, as shown in the next section. This means that employers exploit decreasing marginal returns in order to reduce the wages of each infra-marginal worker. For this reason the term is usually denoted in the literature as the *overemployment* or *overhiring effect*. Alternatively, equation (14) can be interpreted as a labor demand equation which relates the firm's optimal employment and wages choices.

3.4 Wage Bargaining

Let E_j and U denote respectively the asset values of a worker employed in sector j or searching for a job. An unemployed worker enjoys a flow utility z, which may correspond to a combination of home-production and utility flow from leisure enjoyed while unemployed. He has a utility gain of $(E_j - U)$ when he finds a job in sector j, which occurs with probability $\theta_j m_j\left(\theta_j\right)$. Thus, in a steady state we have that:

$$rU = z + \sum_{j=F,I}\theta_j m_j(\theta_j)\left(E_j - U\right) \tag{15}$$

A worker employed in sector j, in his turn, receives a wage w_j and incurs a utility loss of $U - E_j$ when the job is destroyed, which occurs at rate d_j. We then have that:

$$rE_j = w_j + d_j(U - E_j). \tag{16}$$

Workers are not paid their full marginal product as in the standard neoclassical framework due to a combination of costly search and matching frictions which give rise to rent sharing. Most of the papers which study informality incorporating search frictions assume that wages result from Nash bargaining between one worker and one firm in the sector that experiences search frictions. We assume alternatively that bargaining takes place between a firm and its multiple workers, each worker being treated as the marginal worker. This is a good representation of reality when firms cannot commit to long-term contracts and may renegotiate wages with each worker at any time (Mortensen, 2009). This seems an adequate framework to represent a case like the Brazilian one, where job turnover is extremely high (see Gonzaga, 2003).

Furthermore, this interesting alternative assumption has not been implemented yet in the literature studying the composition of employment in terms of formal vs informal jobs. We follow a growing body of the literature that has applied this assumption in studies not related to informality.[13]

As will become clear later, bargaining with multiple workers introduces some important differences compared to the standard one-worker-per-firm framework, as under the neoclassical framework where wages equal marginal product. In particular, it gives rise to a wage bargaining externality according to which firms hire workers above the point where the marginal revenue from hiring the marginal worker equals marginal cost, since by doing so the wage of all inframarginal workers is reduced.[14] In our large-firm setup, the bargaining problem writes:

$$\max_{w_j} \beta_j \ln\left[E_j - U\right] + (1 - \beta_j) \ln \frac{\partial V_j\left(h_j\right)}{\partial h_j}, \tag{17}$$

where $\beta_j \in [0, 1]$ can be interpreted as workers' bargaining power. Then, using equations (13), (16) and the first order condition of (17), it is possible to show that the wage w_j is a solution to the following first-order linear differential equation:

$$w_j(h_j) = (1 - \beta_j)rU + \beta_j \left[\frac{A_j}{1 + \tau_j} \frac{\sigma_j - 1}{\sigma_j} p_j - \frac{\partial w_j(h_j)}{\partial h_j} h_j\right], \tag{18}$$

[13]See, among others, Bertola and Caballero (1994), Stole and Zwiebel (1996), Smith (1999), Cahuc and Wasmer (2001), Delacroix (2006), Cahuc, Marque and Wasmer (2008), Ebell and Haefke (2009), and Felbermayr and Prat (2011).

[14]Our framework is fairly general. It is always possible to compare the situation where that externality is ruled out to the case where it is present, which we do in subsection 5.2. It is also possible to allow for that externality in one sector only.

which has the following solution:[15]

$$w_j(h_j) = (1-\beta_j)rU + \beta_j \frac{\sigma_j - 1}{\sigma_j - \beta_j} \frac{A_j p_j(h_j)}{1+\tau_j}. \tag{19}$$

Equation (19) can be interpreted as the wage curve, which defines the wage as a weighted average of the worker's reservation value rU and of the firm's marginal revenue, captured by the term $\frac{\sigma_j - 1}{\sigma_j - \beta_j} \frac{p_j(h_j)A_j}{1+\tau_j}$.

The relation between wages and employment is clear when one evaluates $\frac{\partial w_j(h_j)}{\partial h_j} h_j$. Combining equations (18) and (19) we get:

$$\frac{\partial w_j(h_j)}{\partial h_j} h_j = - \underbrace{\frac{\sigma_j - 1}{\sigma_j - \beta_j}}_{\text{overhiring and market power effects}} \frac{\beta_j A_j p_j(h_j)}{\sigma_j (1+\tau_j)} < 0. \tag{20}$$

It is clear from equation (20) that wage depends on a term combining overhiring and market power effects. The equation also implies that the bargained wage is a decreasing function of employment. This is due to the fact that the firm's marginal revenue decreases with the number of workers, since the increased production from hiring an extra worker tends to reduce the price p_j. This effect is taken into account by firms enjoying some market power. Given that each worker is treated as the marginal worker, hiring one more worker reduces the wage by $\left| \frac{\partial w_j(h_j)}{\partial h_j} \right|$. This leads to the so-called *overhiring externality*.

Notice that the overhiring effect differs across sectors, since they have different market and bargaining powers. From equation (20), the overhiring externality increases with workers bargaining power β_j and decreases with competition σ_j. It vanishes when $\beta_j \to 0$ or $\sigma_j \to \infty$. In such limit cases, workers are paid a constant wage: they get their reservation wage in the first case, while the marginal product of a worker is constant under full competition in the second. Such limit cases are more likely to apply to the informal sector, where workers' bargaining power is smaller and competition is more intense (see Camargo, 2003).

Combining equations (14) and (20), we get the equation that determines the optimal employment choice:

$$\left[w_j(h_j)(1+\tau_j) + \frac{\gamma_j(r+d_j)}{m_j(\theta_j)(1-\delta_j)} \right] \frac{\sigma_j - \beta_j}{\sigma_j} = A_j p_j(h_j) \frac{\sigma_j - 1}{\sigma_j} \tag{21}$$

According to equation (21), firms set employment, and therefore wages, so as to equalize marginal costs to marginal revenue. Marginal revenue, on the right hand side, includes a factor

[15] Appendix A derives the solution of the differential equation.

$\frac{\sigma_j - 1}{\sigma_j} < 1$, due to their market power in the goods market. Marginal costs, on the left hand side of the equation, consist of wages, taxes on labor and expected search costs (i.e. the overall employment cost). It is weighted by an overhiring factor $\frac{\sigma_j - \beta_j}{\sigma_j} < 1$, which establishes that they set h_j above the efficient level where the benefit from hiring the marginal worker equals his cost. Firms are willing to do so because they are aware that hiring more workers tends to depress wages paid for their entire workforce. In our two-sector setting, overhiring should be less important in the informal sector, where workers' bargaining power β_j is smaller and the elasticity σ_j is larger.[16]

3.5 Equilibrium

We will describe the equilibrium in two steps. First, we derive the equilibrium for an exogenously fixed number of firms in each sector. This equilibrium may be interpreted as a short-run equilibrium, where the number of firms is not necessarily adjusted to the entry incentives. Then, the optimal number of firms is computed given the entry costs in each sector.

3.5.1 Equilibrium for a given number of firms

We start by assuming a fixed number of firms in each sector. In that case, the equilibrium prices p_j, wages w_j, firm size h_j and sectoral employment $L_j = N_j h_j$, aggregate unemployment $u = 1 - \sum_{j=I,F} L_j$, tightness θ_j and workers reservation value are endogenously determined. Note that, given equation 5, having a fixed number of competitors N_j is equivalent to fixing the elasticity σ_j faced by each firm in industry j.

The endogenous variables of our model are all functions of two main unknowns: the labor market tightness in the two sectors (θ_F, θ_I). They are determined by the equilibrium conditions in the goods and in the labor markets as follows. First, using demand and supply equations (3) and (6), we find the employment ratio between the two sectors that satisfies the goods market equilibrium. We denote it the product market equilibrium condition, given by:

$$\frac{L_I}{L_F} = \frac{A_F}{A_I} \left[\frac{p_I(\theta_F, \theta_I)}{p_F(\theta_F, \theta_I)} \right]^{-\sigma}, \tag{22}$$

which defines implicitly the intersectoral allocation of labor as a function of relative prices and relative productivity, a usual property.

[16] Notice that the intuition provided here takes workers' outside options as fixed, and not endogenous as in our model. It turns out that *overhiring* in the formal sector may translate into *underhiring* in the informal sector, which is actually the case in our simulations of the model, as shown in section 5.2.

The relative size of the informal sector L_I/L_F is a function of tightness θ_F and θ_I due to the (positive) dependence of prices p_F and p_I on these variables.[17] A rise in θ_j would imply a rise in wages, translating into a higher price. The quantity consumed thus decreases, resulting in lower employment in the sector.

Note that the impact of tightness θ_F and θ_I on L_I/L_F is, in principle, ambiguous. Assumption 1 guarantees a monotonic relationship between relative size of informality and tightness, as established in Proposition 1.[18]

Assumption 1. *We assume that $\varepsilon_{p_i,\theta_i} > \varepsilon_{p_j,\theta_i}$, where $\varepsilon_{p_i,\theta_j}$ is the elasticity of prices in sector i with respect to labor tightness in sector j, for i and $j = F, I$.*

Proposition 1. *The relative size of the informal sector L_I/L_F is an increasing function of tightness in the formal sector θ_F and a decreasing function of tightness in the informal sector θ_I if, and only if, Assumption 1 is true.*

Proof. Differentiating equation (22), the result is straightforward. □

Second, from equation (9) we derive the relative employment in the two sectors that is compatible with equilibrium in the labor market. We then get the labor market equilibrium condition:

$$\frac{L_I}{L_F} = \frac{d_F}{d_I}\frac{\theta_I m_I(\theta_I)}{\theta_F m_F(\theta_F)}, \tag{23}$$

which defines the intersectoral allocation of labor as a function of sectoral tightness, the efficiency parameter of the matching processes, and turnover rates. Hence, the labor market equilibrium condition implies that the informal sector is relatively larger when its own labor market tightness θ_I is higher and when the formal sector's tightness θ_F is lower. Moreover, the formal sector is also larger when its own destruction rate of jobs is lower and the informal sector's one is higher.

The intersectoral allocation of labor in equilibrium is determined when product and labor market equilibrium relationships are satisfied simultaneously. Hence, by equalizing equations (22) and (23) we determine the Intersectoral Allocation of Labor curve (IALC):

$$\frac{d_F}{d_I}\frac{\theta_I m_I(\theta_I)}{\theta_F m_F(\theta_F)} = \frac{A_F}{A_I}\left[\frac{p_I(\theta_F,\theta_I)}{p_F(\theta_F,\theta_I)}\right]^{-\sigma}, \tag{IALC}$$

[17] Appendix B shows how prices can be written as a function of labor market tightness in the two sectors.

[18] We argue Assumption 1 is a reasonable assumption. From equation (30) in appendix B, we see that there is a direct impact of labor tightness on its own sector prices, while the impact on the other sector prices is indirect, through its impact on the value of being unemployed U.

which determines a first relationship between θ_F and θ_I. The IALC condition establishes implicitly a positive relationship between θ_F and θ_I if the conditions on price elasticities in proposition 1 are met.

A second relationship between the two sectoral tightnesses is obtained using the aggregate price index and normalizing it to one:

$$P = 1 = \alpha^{\frac{1}{1-\sigma}} \left[p_I(\theta_I, \theta_F)^{1-\sigma} + p_F(\theta_I, \theta_F)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

We get the Price Curve (PC):

$$1 - \alpha p_I(\theta_I, \theta_F)^{1-\sigma} = \alpha p_F(\theta_I, \theta_F)^{1-\sigma}, \tag{PC}$$

which defines a decreasing relationship between θ_I and θ_F.

The IALC and PC relationships together determine the equilibrium levels of labor market tightness in the two sectors, θ_F^* and θ_I^*, by means of a fixed point argument.

The equilibrium is unique provided the IALC relationship evolves monotonically with respect to θ_I and θ_F, as represented in Figure 5. This monotonicity is assured when Assumption 1 is satisfied, which is the case in our numerical simulations.[19]

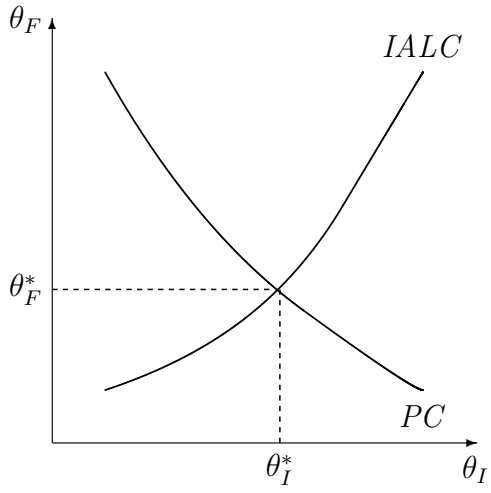

Figure 5: Unique Equilibrium

Having determined equilibrium values for labor market tightness θ_I^* and θ_F^*, all the other variables of the model follow. Note that formal and informal sectors are interdependent for

[19]Notice that existence is guaranteed provided z and τ are not too large compared to productivity A. This can easily be seen in the simple case where the two sectors are identical. In such a case, the IALC becomes the 45° line in Figure 5 ($\theta_I = \theta_F$) but intersects with PC iff $A \geq \frac{\sigma N - \beta}{\sigma N - 1}(1 + \tau)z$.

basically two reasons : (i) in equilibrium, demand for goods and therefore the relative size of sectors depend on relative prices, as is clear from equation (22); (ii) workers search for jobs in both sectors, as established by equation (15). As a result, a change in sector-specific parameters affects both sectors, as will be shown in the next section.

3.5.2 Equilibrium number of firms

The next step is to determine the number of firms in each industry in equilibrium. The timing of events is the following. At the beginning of a period firms decide whether to enter the market. If they enter they pay an entry cost, c_j, on top of the cost of posting vacancies in a number sufficient to recruit the desired amount of workers.[20] Business is then started and profits are received at the end of that period/beginning of the next period.

Entry costs entail direct administrative costs as well as indirect costs due to administrative delays. Several of the entry costs do not apply to the informal sector, such as, for instance, the official registration to comply with legislation. Although it would be fair to say that barriers to entry are essentially a problem in the formal sector, informal firms may still incur entry costs since it may take some time and resources to set up a business in this sector. It is then reasonable to assume that $0 \leq c_I < c_F$.

In equilibrium, the free entry condition establishes that the costs of setting up a business must equal its profits, as in:

$$c_j + \frac{\gamma_j h_j}{m_j(\theta_j)} = \frac{1+r}{r+\delta_j}\pi_j \tag{24}$$

where the second term in the left-hand side corresponds to the cost of posting vacancies to hire the desired amount of labor h_j, and π_j stands for profits, which is given by:

$$\pi_j = p_j A_j h_j - w_j h_j(1+\tau_j) - \gamma_j s_j h_j / m_j(\theta_j). \tag{25}$$

We have derived all variables as functions of labor market tightness θ_I and θ_F, which are themselves functions of the number of firms operating in each sector, N_I and N_F. Hence, in equilibrium, all variables are defined as functions of N_I and N_F and equation (24) closes the model .

Firms' profit opportunities decline with the number of firms operating in the market N_j since with more firms there is more competition and lower markups. Under free entry, a rise

[20]Notice that, in our setup, firms jump to their steady state size when they enter the market. This is a consequence of our assumption of linearity of adjustment costs. See Bertola and Caballero (1994) for a model with convex costs. See also Acemoglu and Hawkins (2010) for an alternative framework where firms cannot hire a large number of workers in each period.

in the left hand side of the free entry condition (24), for instance, due to an increase in entry costs c_j or in hiring costs $\frac{\gamma_j h_j}{m_j(\theta_j)}$ must be compensated by an equal rise of the right hand side of (24), i.e. higher profits, which is obtained by a smaller equilibrium number of firms.

4 Quantitative analysis

We now proceed to the quantitative analysis of our model. Our aim is to show that the main endogenous variables of the model match the stylized facts described in section 2, and then to study how they respond to policy changes.

4.1 Calibration

We choose parameters with two criteria in mind: (i) they have to be realistic and coherent with the values usually used in the literature, (ii) the values of endogenous variables stemming from the simulations have to be realistic and/or comparable with the values found in previous studies. We choose the Brazilian economy to guide our parametrization. Brazil is a large developing country with a sizable informal sector, with the advantage of having high quality micro data available, which has already been exploited in a number of empirical studies. Hence, we have access both to data and to other studies that have worked on them.

Our reference period is a month and we use 2003 as reference year. The discount rate r is set to 0.6434% which corresponds to an annual rate of 8% as in Heckman and Pagés (2003). All relevant variables and parameters are allowed to differ between the formal and the informal sectors. Informal sector firms are assumed to be less productive than formal ones, and their productivity is normalized to one. The productivity differential between the formal and the informal sector is large and consistent with the range of estimates provided by La Porta and Shleifer (2008). The productivity parameter in the formal sector is 2, capturing a productivity differential of 100%, as used by other studies (see, for instance, Ulyssea, 2009). In terms of our notation, we then have that $A_F = 2$ and $A_I = 1$.

In a recent study, Bartelsman, Haltiwanger, and Scarpetta (2009) estimate that the annual exit rate for Brazilian firms ranges between 5% to 10%, and they indicate that the exit rate is higher among smaller firms. Since firms are on average smaller in the informal sector than in the formal one, we use the lower boundary of the interval to define firms' exit rate in the formal sector and, conversely, the upper boundaary to define firms' exit rate in the informal sector. It follows that the monthly values for the two parameters are set to $\delta_F = 0.0041$ and $\delta_I = 0.0080$

which is consistent with the intuition that on average firms' turnover is higher in the informal sector, i.e. $\delta_F < \delta_I$. Labor turnover is higher in the informal sector $d_F < d_I$. We choose the parameters to be equal to $d_F = 0.0221$ and $d_I = 0.0102$ which correspond to an annual rate of 13% and 30% as in Heckman and Pagés (2003) and Ulyssea (2009). Finally, making use of equation (7), the exogenous separation rates are set to $s_F = 0.0062$ and $s_I = 0.0142$ respectively.

Matching functions are of the Cobb-Douglas form, i.e. $M_F = \kappa_F u^{\eta} v_F^{1-\eta}$ and $M_I = \kappa_I u^{\eta} v_I^{1-\eta}$. The elasticity of the matching function is set to one half, as usual in the literature (Petrongolo and Pissarides, 2001, Shimer, Rogerson and Wright, 2005) while the scale parameter of the matching function is set to target an unemployment rate of approximatively 12.5% as in the data (See section 2). According to Camargo (2005) the bargaining power in the informal sector is approximately 1/3 of that in the formal sector. Those parameters can then be set to $\beta_F = 0.45$ and $\beta_I = 0.15$, as in Ulyssea (2009). By definition, the labor tax rate is nil in the informal sector and we set the formal tax rate equal to 30% which is consistent with the value reported in the World Doing Business Indicators for social security contributions and payroll taxes.

Keeping in mind that firms in informal sectors face lower (if any) entry and flow costs ($c_F > c_I$ and $\gamma_F > \gamma_I$), we set the remaining free parameters so as: (i) to replicate the size of the informal sector; (ii) to get a reasonable wage premium ($w_F > w_I$); (iii) to have a faster (more efficient) matching process in the informal sector ($M_F < M_I$); (iv) to have more firms in the informal sector ($N_F < N_I$) but with a smaller size ($h_F > h_I$). Baseline parameters are reported in Table 2.

4.2 Benchmark results

The benchmark results of our numerical exercise are summarized in Table 3.[21] They match our targets in terms of aggregate variables, with an unemployment rate around 12% and an informal sector representing 40% of total employment. Wages are approximately 18% higher in the formal sector compared to the informal one, which is consistent with the estimation of the wage differentials between the two sectors, as presented in Figure 3, section 2. Very similar patterns can be found in Bargain and Kwenda (2009) and Tannuri-Pianto and Pianto (2002) for quantile regressions.

The job finding rate is two times higher in the informal than in the formal sector. Although

[21] We have checked that the PC and IALC equilibrium condition have a graphical representation in the θ_F, θ_I space as depicted in Figure 5.

Table 2: Baseline parameters

Parameter	Sector		Informal/Formal	Description
	Informal	Formal		
A_j	1	2	0.50	Productivity
δ_j	0.80%	0.41%	1.95	Firms' exit rate
d_j	2.21%	1.02%	2.16	Matches' dissolution rate
γ_j	0.45	1.10	0.41	Cost of a vacancy
κ_j	0.0500	0.0475	1.05	Matching function parameter
β_j	0.15	0.45	0.33	Workers' bargaining power
τ_j	0	30%	-	Tax on labor costs
c_j	0.50	2.25	0.22	Entry cost
General parameters				
r		0.65%		Discount rate
σ		2.05		Elasticity of substitution
α		0.5		Utility function parameter
z		0.05		Workers' reservation value

Table 3: Main endogenous variables

Variable	Sector		Informal/Formal	Description
	Informal	Formal		
w_j	0.77	0.91	0.85	Wage
θ_j	2.18	0.69	3.16	Labor market tightness
n_j	5.55	3.12	1.78	Number of firms
h_j	0.07	0.15	0.47	Firm size
σ_j	11.37	6.40	1.78	Demand elasticity for a firm
π_j	0.02	0.05	0.04	Profits
u	12.17			Unemployment rate
L_I	40.67			Informality rate

this difference is somewhat larger than the one found in Table 1 of section 2, it is consistent with the viewpoint taken in most existing studies where finding a job in the informal sector is easier than in the formal one (see, e.g. Zenou, 2008, Ulyssea, 2009).

Also consistent with the evidence, there are fewer and larger firms in the formal than in the informal sector as argued by Rauch (1991) and discussed in Tybout (2000) and La Porta and Shleifer (2008). More precisely, we find formal firms to be approximately two times larger than informal ones. Correspondingly, informal firms are approximately two times more numerous in the informal sector. The resulting price elasticity of demand is around 6 in the formal sector compared to about 11 in the informal one. As a consequence, profits are higher in the formal sector. Having different values for the elasticity is a desirable feature of our model. It implies that the various externalities stemming from market and bargaining powers are of different magnitude across sectors.

4.3 Policy Analysis

4.3.1 Product market regulation[22]

We start by analyzing the impacts of a change in product market regulation, which is captured by the entry costs in the formal sector c_F. It represents changes such as simplifying procedures and reducing taxes related to setting up a new business.

As shown in Figure 6, a decrease in entry costs c_F leads to (i) higher wages in both sectors, with a decrease of the informal sector relative wage (ii) a smaller relative size of the informal sector, (iii) lower unemployment (iv) an increase in the degree of competition for formal firms compared to informal ones. In quantitative terms, as a consequence of a 1 percent decrease in informality, unemployment would fall by 0.15 percent and the informal sector relative wage would fall by 0.1 percent. This is induced by a 15 percent decrease in entry costs.

Figure 6 about here

The intuition for the results is the following. A reduction in formal sector entry costs increases the number of formal sector firms, while it reduces the number of informal ones. As a result, price elasticities increase for formal sector firms and decrease in the informal sector. The effect on relative competition across sectors is thus magnified, as shown by the change in $\frac{\sigma_I}{\sigma_F}$ in

[22]We here focus on the long run equilibrium where the number of firms is made endogenous. Alternatively, we might have chosen to consider the short run equilibrium where the number of firms is exogenous, and to let the number of firms vary to proxy fiercer competition among formal firms. The two scenarios yield almost the same qualitative results.

Figure 6. Fiercer competition in the formal sector induces firms to charge lower prices, which increases the demand for formal sector goods. The size of the formal sector thus increases and unemployment decreases.

With more vacancies posted in the formal sector, the probability for an unemployed person to find a formal job increases, which in turn raises the return to search and increases workers' reservation utility. Wages rise, not only in the formal sector but also in the informal one, as a result of an integrated labor market. The impact on formal wages is nevertheless stronger, resulting in a higher formal sector wage premium.

Overall, if the formal sector were more similar to the informal one in terms of entry costs, its degree of competition would increase. Aggregate employment would then be slightly higher while the informal sector would represent a significantly lower share of total employment. Wages would increase in both sectors, with a relatively larger increase in the formal sector.

Getting back to what we have seen in section 2, the Brazilian economy has experienced a simultaneous reduction in informality and in unemployment over the past decade. These movements, however, were accompanied by an increase in informal sector relative wages. Hence, a decrease in formal sector entry costs cannot fully account for the pattern of changes of Brazilian variables, even though product market regulation declined over the period.[23]

4.3.2 Labor market regulation

We take the bargaining power of workers in the formal sector as an indicator of labor market regulation. Bargaining power may be thought of as a variable that captures broad institutions that affect the functioning of the labor market and allow workers to extract rents.[24] Figure 7 displays how the economy reacts to an exogenous change of workers' bargaining power in the formal sector. The impact of changing workers' bargaining power is a combination of its direct impact and its impact on the number of firms in each sector.

The effect of a decrease in formal workers' bargaining power is unambiguously (i) a decrease in wages in both sectors, but to a lower extent in the informal sector, (ii) a decrease in unemployment, (iii) a decrease in informal relative to formal employment and (iv) an increase in the degree of competition for formal firms compared to informal ones. In terms of magnitudes, as a result of a 1 percent decrease in informality, unemployment would fall by 1.49 and the informal

[23]See for instance the World Bank Doing Business indicators for 'Starting a Business' at http://www.doingbusiness.org/Custom-Query/brazil.

[24]Similar interpretations are made in e.g. Blanchard and Giavazzi (2003) or Ebell and Haefke (2009).

sector relative wage would increase by 0.82 percent. This results from a 3 percent decrease in workers' bargaining power.

Figure 7 about here

A reduction in workers' bargaining power in the formal sector increases profits in that sector, which attracts more firms to it. The number of formal firms increases, more vacancies are posted in the sector, thereby formal sector employment increases. On the other hand, more vacancies in the formal sector increases the probability that an unemployed individual finds a formal job, which would tend to increase wages. Nevertheless, the direct impact of lower bargaining power is stronger, and formal wages decrease. Workers' reservation utility then decreases, causing a reduction in informal sector wages as well, through the integrated labor market, although of smaller magnitude.

All other things being equal, lower informal wages increase profits, attracting more firms to informality. More vacancies are posted in the informal sector as well, but the effect is of smaller magnitude compared to the formal sector. The fact that there are more matches overall yields a stronger reduction of unemployment compared to the case of a reduction in formal sector entry costs. The composition effect on firms' entry into both sectors decreases the relative competition in the informal sector, as shown in Figure 7.

Overall, it turns out that if the formal sector were more similar to the informal one in terms of bargaining power (that is, a lower β_F, due to a cutback in labor market regulation, for instance), then unemployment would be lower, and the informal sector would represent a lower share of total employment. Moreover, wage inequality would decrease. Getting back to the Brazilian case, this set of effects is compatible with the stylized facts detailed in section 2: unemployment, informality and the wage premium have decreased over the period.

4.3.3 Fiscal policies

Fiscal policies may be used as instruments to reduce informality. Lower taxation would attract more firms to the formal sector, reducing informality in the economy. In terms of our model, this translates into a reduction of τ_F. Alternatively, a more direct way to reduce informality would be simply to increase both supervision to detect informal firms and enforcement for them to comply with formality. Such a policy may be captured in our model by an increase in the exit rate of informal firms δ_I, on the assumption that informal firms cannot bear either penalties from being caught or formality costs. We analyze these two policy changes in turn.

Taxes As shown in Figure 8, a decrease in formal sector taxation (i) increases wages in both sectors, but to a higher extent in the formal sector, (ii) raises unemployment, and (iii) decreases the relative size of the formal sector. Finally, it (iv) renders the degree of competition in both sectors slightly more similar. Namely, in order to achieve a 1 percent decrease in informality, unemployment would rise by 0.2 and the informal sector relative wage would fall by 0.3 percent. This is accomplished through a 6 percent reduction in labor taxes.

Figure 8 about here

The impact of lowering taxes is very similar to that of decreasing entry costs. Both policies increase formal sector profits, which impacts wages and the relative size of the sector through similar mechanisms. An important difference between the two policies lies in their impact on unemployment. While unemployment decreases with lower entry costs in the formal sector, it increases with lower taxes. This difference is due to the fact that lowering entry costs has a stronger (positive) impact on the number of firms in the formal sector. Lower entry costs attract relatively more firms to the formal sector, which more than compensates for the drop in informal employment. Comparing the last graph of Figures 6 and 8, we see that the impact on the relative degree of competition between the two sectors is stronger for changes in entry costs c_F than for changes in taxes τ_F.

Enforcement We turn to the impact of increasing enforcement of formality, which is represented in our model by an increase in informal firms' exit rate δ_I. As shown in Figure 9, the overall result of a rise in the exit rate of informal firms is (i) a reduction in wages in both sectors, with a larger drop in the informal sector, (ii) an increase in unemployment, (iii) an informality reduction, (iv) and a lower degree of competition in the informal sector with respect to the formal one. In terms of quantities, as a result of a 1 percent decrease in informality, unemployment would rise by 0.35 percent and the informal sector relative wage would fall by 0.13 percent. Such a quantitative effect is induced by a 12 percent rise in enforcement.

Figure 9 about here

From equation (10), it is evident that a higher exit rate reduces the expected value of informal firms, which has negative impact on informal sector wages. On top of that, fewer firms are attracted to the sector, reducing both the number of firms and vacancies. The resulting lower contact rate decreases the returns to searching, which decreases wages in both sectors,

with a mechanism similar to that of previous exercises. Clearly, the negative impact on informal sector wages is stronger than on wages in the formal sector, since the latter are affected only indirectly through workers' reservation utility.

Formal sector firms earn higher profits due to the wage reduction. The number of formal firms increases, rendering the formal sector more similar to the informal one in terms of degree of competition. Unemployment, in turn, always increases with δ_I, meaning that the rise in formal employment is not sufficient to outweigh the fall in informal employment. With respect to the number of operating firms, it increases in the formal sector and decreases in the informal sector. Finally, relative price elasticity $\sigma_I/\sigma_F = N_I/N_F$ is negatively affected by the rise in δ_I .

5 Counterfactual experiments

In this section, we run a number of counterfactual exercises. Our aim is twofold: (i) we seek to shed some light on the relative impact of the main features of our model; (ii) in addition, we wish to evaluate the robustness of our results. For this purpose, we first investigate the properties of this economy if all firms were to go formal. Second, we assess the impact of the three main sources of market imperfection at play in our model: the wage bargaining and matching frictions in the labor market, and imperfect competition in the product market. Third, we discuss the impact of segmented versus unsegmented search in this economy, and lastly, the relationship between firms' size and market power.

5.1 No informality

It has been argued that the informal sector may serve as an alternative for entrepreneurs to escape the burden of excessive regulations and costs in the formal sector. The main argument is that, without the option of informality, unemployment would be much higher. The corollary of this view is that informality should not be fought, unless accompanied by a reduction in the costs and rigidities associated with formality. We examine this argument by simulating the model assuming that both sectors are formal, that is, we assume both sectors share the formal sector parameters in Table 2.

The first two columns of Table 4 present the results for an economy with no informality, where the only difference between the two sectors is their productivity; that is, we still have a low and a high productivity sector. Effectively, unemployment does increase, but the rise is relatively modest: about two percentage points. With no informality, the difference between

Table 4: Counterfactual: no informality

Variable	Sector: Informal $A_I = 1$	Sector: Formal $A_F = 2$	Symmetric cases $A = 2$	Symmetric cases $A = 1$	Description
p_j	1.39	0.78	1.00	1.00	Price
w_j	0.79	0.86	1.16	0.52	Wage
θ_j	0.25	0.64	0.66	0.25	Labor market tightness
n_j	2.40	3.21	3.30($\times$2)	2.28($\times$2)	Number of firms
h_j	0.14	0.16	0.13	0.18	Firm size
σ_j	4.92	6.57	6.77	4.69	Elasticity of demand for a firm
π_j	0.04	0.06	0.05	0.05	Profit
u	14.26		11.73	17.72	Unemployment rate

the two sectors decreases in all dimensions, as expected. It is interesting to note that the number of firms in the low productivity sector is now smaller than in the high productivity sector, and the same is true for labor market tightness. When informal, the low productivity sector becomes more attractive since it is able to escape from regulations and labor market rigidities associated with formality.

There are also advocates of fighting informality no matter what, on the grounds that informality is in itself a source of low productivity. Formal firms, for instance, have access to the judiciarl system and can write enforceable contracts, which is beneficial in a number of ways. In particular, they have access to credit markets, which allows them to invest not only in capital to achieve an optimal scale of production, but also in research and development to increase total factor productivity. In sum, the idea is that when firms become formal their productivity increases. In the extreme case where informal firms become as productive as formal ones when they turn formal, unemployment would actually decrease slightly, as shown in the third column of Table 4. This more productive economy would pay higher wages and be more competitive than the one with a low productivity sector.

Notice that if the whole economy had the productivity of the disadvantaged sector, unemployment would be significantly higher and wages lower, as shown in the fourth column of Table 4. Actually, to keep the unemployment rate unchanged when formalizing the whole economy, the productivity of the informal sector would have to rise by 76%, which is a non-negligible increase.

5.2 The role of market imperfections

We investigate the role of each market imperfection at play in our model: wage bargaining, matching frictions and monopoly power in the product market. For this purpose, we simulate the model under different intensities of each market imperfection. More precisely, we perform three exercises. In the first exercise, we rule out the wage bargaining externality by imposing $\frac{\sigma_j-\beta_j}{\sigma_j}=1$ in both sectors. In a second exercise, we consider a less frictional labor market by a 10% decrease in the scale parameter of the matching function κ_j, also in both sectors. Finally, we consider a more competitive goods market by a 10% increase in σ. Table 5 presents the percentage changes with respect to the benchmark case for each scenario. With these exercises, we seek to shed some light on the relative impact of each externality of our model. In addition, they are also useful to evaluate the robustness of our results.

Table 5: The role of market imperfections (% changes w.r.t the benchmark case)

Market imperfection	u	$\frac{L_I}{L_F}$	$\frac{w_I}{w_F}$
No overhiring	0.84	2.96	0.19
Fewer matching frictions	−10.25	−0.46	1.63
Less monopoly power	−2.10	−0.06	0.28

Wage bargaining We first compare the results obtained in the benchmark case to those obtained when the wage bargaining externality is ruled out in the two sectors. First, as shown in the first line of Table 5, it turns out that unemployment would be higher in the absence of the wage bargaining externality, compared to the benchmark case with overhiring, as expected. Ruling out the wage bargaining externality would cause an increase in unemployment of the order of 0.8%, which is quite modest. Nevertheless, that externality has a relatively stronger impact on the relative size of the informal sector. More specifically, L_I/L_F is about 3% larger when the externality is ruled out.[25] Finally, informal firms pay lower relative wages with the wage bargaining externality compared to without, but the difference between the two cases is quite small.

Matching frictions We now compare the results obtained in the benchmark case to those obtained when the formal and informal labor markets experience a 10% improvement in the

[25]Tightness is smaller in the informal sector and larger in the formal sector in the presence of the wage bargaining externality, i.e. informal firms tend to *underhire*, while formal firms *overhire*, leading to a smaller relative size of the informal sector.

Table 6: Segmented labor market

Variable	Sector		Description
	Informal	Formal	
w_j	0.47	0.99	Wage
θ_j	4.12	1.00	Labor market tightness
n_j	5.70	2.78	Number of firms
h_j	0.09	0.12	Firm size
u	17.83		Unemployment rate

matching efficiency parameter κ_j. The results are reported in the second line of Table 5. First, we see that unemployment is lower by approximately 10% when labor markets become less frictional. Second, the relative size of the informal sector L_I/L_F is reduced, as expected. Namely, matching frictions provide a rationale for the existence of the informal sector: if it took no time to locate a job offer in the formal sector, there would not be an informal sector. Therefore, the underlying assumption for the existence of the two sectors is that they are sufficiently frictional. Third, relative wages w_I/w_F increase when matching frictions are reduced.

Market power In the last exercise, we increase the elasticity of substitution between formal and informal goods σ by 10% to analyze the role of market power in our model. The results are presented in the third line of Table 5. First, it turns out that lowering market power by 10% would decrease unemployment by a little more than 2%. Second, the impact of a reduction in market power on the relative size of the informal sector is quite small.[26] Finally, less market power means lower relative wages in the informal sector.

5.3 Labor market segmentation

Our model takes the labor market as perfectly integrated, whereas other studies have considered a segmented labor market. We then investigate how our results would change if the labor market were fully segmented, as in e.g. Fugazza and Jacques (2003). More precisely, we assume that a proportion ϕ of the unemployed searches for informal jobs exclusively while the remaining 1-ϕ searches in the formal sector only. Table 6 depicts the values of the main endogenous variables of our model for $\phi = 0.6$.

[26]Actually, when we simulate the model for alternative values of bargaining power and entry costs we find that when bargaining power is high or when entry costs are low, lessening market power increases informality. The opposite is true for low bargaining power or high entry costs.

The main differences between the results of the model with labor market segmentation in Table 6 and those obtained in the benchmark case with no segmentation in Table 3 are the following. First, unemployment would be considerably larger with segmented markets, since workers' exit rate from unemployment would fall from $\theta_I m_I(\theta_I)+\theta_F m_F(\theta_F)$ for all job seekers to $\theta_I m_I(\theta_I)$ and $\theta_F m_F(\theta_F)$ for those searching in the informal and formal sector respectively. Second, labor market segmentation would increase wages in the formal sector, and cause a substantial drop in informal sector wages. The reason is that, when the labor market is integrated, the higher formal wages increase the fall-back utility of all job seekers, which bids up informal sector wages. The converse is true for wages in the formal sector. As a result, the formal sector wage premium tends to be larger when formal and informal labor markets are segmented.

5.4 Firm size and market power

Differences in firm size play an important role in the determination of informality, unemployment and product market competition in the two sectors. To illustrate this idea, Table 7 below presents the results of two counterfactual exercises where we set exogenously the size of firms to be the same in the two sectors: in the first two columns, we set the size of all firms as the size of informal firms in the benchmark case, while in the last two columns, all firms have the size of formal firms in the benchmark.

In the first case, formal firms are smaller and more numerous compared to the benchmark. Therefore, the formal sector would be more competitive and formal job creation would increase compared to the benchmark. In addition, this change in formal firm size would also feed back to the informal sector where firms would become slightly less numerous, resulting in a less competitive informal sector, with less informal job creation. Overall, informality and unemployment would decrease.

Symmetrically, in the second case informal firms are larger and less numerous compared to the benchmark. The informal sector would be much less competitive, and job creation in that sector would be lower compared to the benchmark case. That would also impact the formal sector, where firms would be slightly more numerous. Product market competition would therefore be somewhat more intense, resulting in higher job creation in the formal sector. Overall, informality would be reduced compared to the benchmark, but unemployment would rise.

Table 7: Identical size of firms

Variable	Sector		Sector		Description
	Informal	Formal	Informal	Formal	
	$h_I = h_F = 0.07$		$h_I = h_F = 0.15$		
w_j	1.04	1.24	0.68	0.83	Wage
θ_j	1.69	1.14	1.10	0.70	Labor market tightness
n_j	5.02	5.00	3.29	3.34	Number of firms
u	11.24		13.75		Unemployment rate
L_I	33.08		32.68		Informality rate

6 Conclusion

The informal sector constitutes an important part of the economy in developing countries, employing in some cases more than half of the labor force. In such an environment, we take the view that the informal sector is not a residual sector and therefore faces the same type of labor and product market imperfections as the formal one. We propose a comprehensive model economy in this perspective, and we investigate the impacts of a number of policy measures that reduce informality.

We build a unified framework in which firms in both sectors face matching frictions in the labor market, monopoly power in the goods market, and engage in wage bargaining. A noteworthy feature of our model is that we assume that firms are large and they choose their size endogenously. The degree of competition among firms in each sector is thereby an endogenous variable. The simulated model replicates the main features of a developing economy with a large informal sector.

We study, alternatively, the impacts of labor and product market deregulation, lowering taxes paid by formal firms and increasing enforcement. We find that both regulatory policies simultaneously decrease informality and unemployment, indicating that there is not necessarily a tradeoff between informality and unemployment. The tradeoff appears when fiscal policies are used though: lower taxes and higher enforcement reduce informality but they increase unemployment.

When inequality is already high, one should be cautious about further increasing it. With respect to their impact on wages, we find that product market deregulation and lower taxes increase wages but they also increase wage inequality. Labor market deregulation is the only policy that decreases wage inequality, but it also reduces wages in both sectors. Increasing

enforcement is the least favorable policy, since it reduces wages, increases wage inequality and raises unemployment, although it is efficient in reducing informality.

A number of developing economies with large informal sectors have undergone significant trade liberalization policies. It may be argued that the effects of trade liberalization are qualitatively the same as those of a product market deregulation. We believe, though, that further research in this area is called for, with the development of a full-fledged open economy model with an informal sector. This is on our research agenda.

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A Wage equation

The FOC to (17) writes :

$$(1-\beta_j)(E_j-U)=\frac{\beta_j}{1+\tau_j}\frac{\partial V_j(h_j)}{\partial h_j}. \tag{26}$$

Using the steady-state condition $\frac{\partial V_j(h_j)}{\partial h_j}=\frac{\partial V_j\left(h_j'\right)}{\partial h_j'}=\frac{\gamma_j}{m_j(\theta_j)(1-\delta_j)}$, and the envelope condition (13) we get:

$$\frac{\partial V_j(h_j)}{\partial h_j}=\frac{1}{r+d_j}\left[A_j\frac{\sigma_j-1}{\sigma_j}p_j-\left(w_j(h_j)+\frac{\partial w_j(h_j)}{\partial h_j}h_j\right)(1+\tau_j)\right] \tag{27}$$

Substituting equations (27) and (16) into equation (26) we get:

$$(1-\beta_j)\frac{w_j - rU}{r+d_j} = \frac{\beta_j}{(r+d_j)(1+\tau_j)}\left[A_j\frac{\sigma_j-1}{\sigma_j}p_j - \left(w_j(h_j) + \frac{\partial w_j(h_j)}{\partial h_j}h_j\right)(1+\tau_j)\right]$$

or

$$\frac{\partial w_j(h_j)}{\partial h_j} + \frac{1}{\beta_j h_j}w_j - \frac{(1-\beta_j)rU + \beta_j\frac{\sigma_j-1}{\sigma_j(1+\tau_j)}p_jA_j}{\beta_j h_j} = 0 \tag{28}$$

which defines the wage w_j as a solution to a differential equation of the form $\frac{\partial w_j}{\partial h_j} + F(h_j)w_j(h_j) + G(h_j) = 0$, with $F(h_j) = \frac{1}{\beta_j h_j}$ and $G(h_j) = -\frac{(1-\beta_j)rU + \beta_j\frac{\sigma_j-1}{\sigma_j(1+\tau_j)}p_jA_j}{\beta_j h_j}$. Equation (28) admits as a solution

$$w_j(h_j) = \left[K - \int_0^{h_j}\frac{G(\zeta)}{H(\zeta)}d\zeta\right]H(h_j) \tag{29}$$

where $H(.)$ solves the homogeneous equation $dH/dh_j + F(h_j)H(h_j) = 0$ which can be rewritten

$$\frac{dH/dh_j}{H(h_j)} = -F(h_j) \text{ or } h_j\frac{dH/dh_j}{H(h_j)} = -1/\beta_j$$

Thus

$$H(h_j) = h_j^{-1/\beta_j}$$

As in Cahuc and Wasmer (2001), we assume the wage w_j is bounded at $h_j = 0$, which implies $K = 0$ in equation (29). w_j can then be rewritten as:

$$\begin{aligned} w_j(h_j) &= h_j^{-1/\beta_j}\int_0^{h_j}\zeta^{1/\beta_j-1}\left[\frac{(1-\beta_j)}{\beta_j}rU + \frac{\sigma_j-1}{\sigma_j(1+\tau_j)}p_j(\zeta)A_j\right]d\zeta \\ &= (1-\beta_j)rU + \beta_j\frac{\sigma_j-1}{\sigma_j(1+\tau_j)}h_j^{-1/\beta_j}A_j\int_0^{h_j}\zeta^{1/\beta_j-1}p_j(\zeta)d\zeta \end{aligned}$$

Integrating by parts and using the fact that $dp_j/d\zeta = -p_j/\sigma_j\zeta$ from the inverse demand function, we have $\int_0^{h_j}\zeta^{1/\beta_j-1}p_j(\zeta)d\zeta = \frac{\sigma_j}{\sigma_j-\beta_j}h_j^{1/\beta_j}p_j(h_j)$. This leads to equation (19).

B Price equation

Substituting equations (19) and (20) into equation (14) we get:

$$p_j = \frac{1}{A_j}\frac{\sigma_j-\beta_j}{\sigma_j-1}\left[rU(1+\tau_j) + \frac{\gamma_j(r+d_j)}{m_j(\theta_j)(1-\delta_j)(1-\beta_j)}\right] \tag{30}$$

which determines the optimal price set by firms as a function exclusively of variables exogenous to the firm's decision.

Then, substituting the Nash Bargaining equation (26) into workers' reservation wage in equation (15), and using the optimality condition $\frac{\partial V_j}{\partial h_j} = \frac{\gamma_j}{m_j(\theta_j)(1-\delta_j)}$, we get:

$$rU = z + \sum_{j\in\{I,F\}} \theta_j m_j(\theta_j) \frac{\beta_j}{(1+\tau_j)(1-\beta_j)} \frac{\partial V_j}{\partial h_j} \tag{31}$$

$$= z + \sum_{j\in\{I,F\}} \frac{\beta_j}{(1+\tau_j)(1-\beta_j)} \frac{\gamma_j \theta_j}{1-\delta_j} \tag{32}$$

Finally, substituting (32) in (30) yields :

$$p_j = \frac{1}{A_j} \frac{\sigma_j - \beta_j}{\sigma_j - 1} \left[(1+\tau_j) z + \frac{1+\tau_j}{1+\tau_k} \frac{\beta_k}{1-\beta_k} \frac{\gamma_k \theta_k}{1-\delta_k} + \frac{r + d_j + \theta_j m_j(\theta_j)}{(1-\delta_j)(1-\beta_j)} \frac{\gamma_j}{m_j(\theta_j)} \right] \tag{33}$$

where $j, k \in \{I, F\}, k \neq j$. With some abuse of notation, this can be written $p_j(\theta_j, \theta_k)$, an increasing function of its two arguments, θ_j and θ_k.

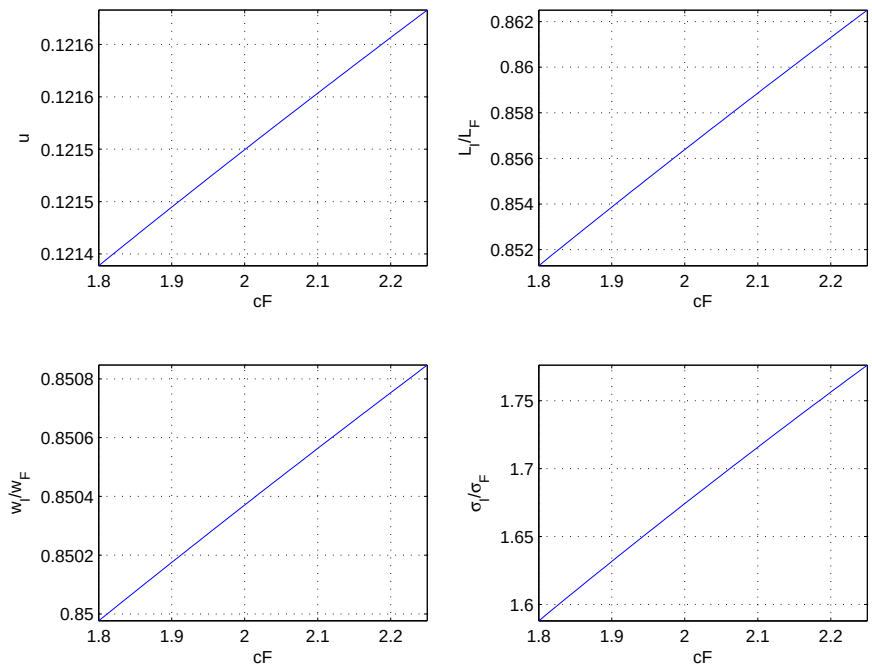

Figure 6: The long-run impact of varying barriers to entry in the formal sector

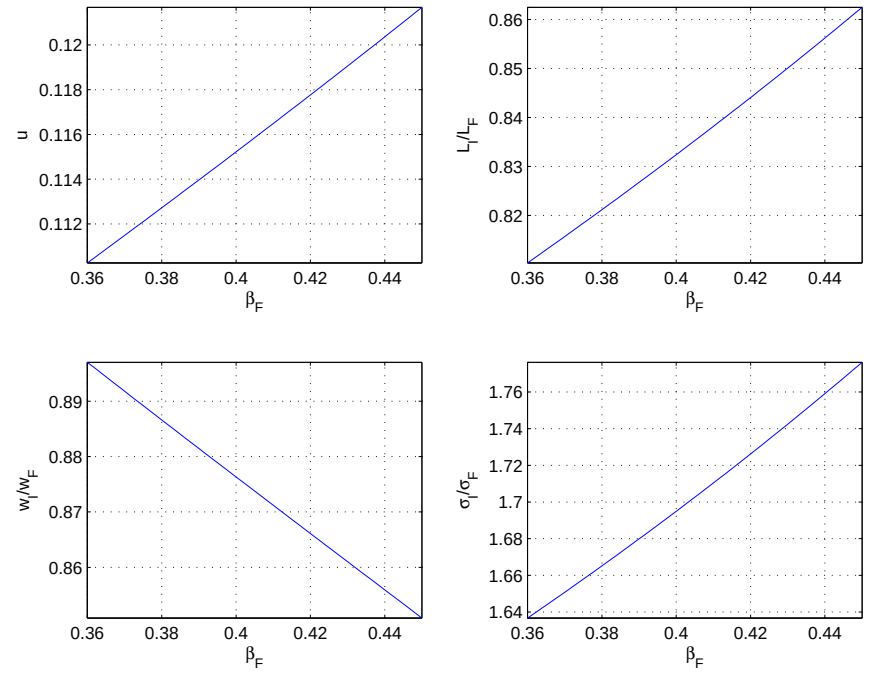

Figure 7: The long-run impact of varying bargaining power in the formal sector

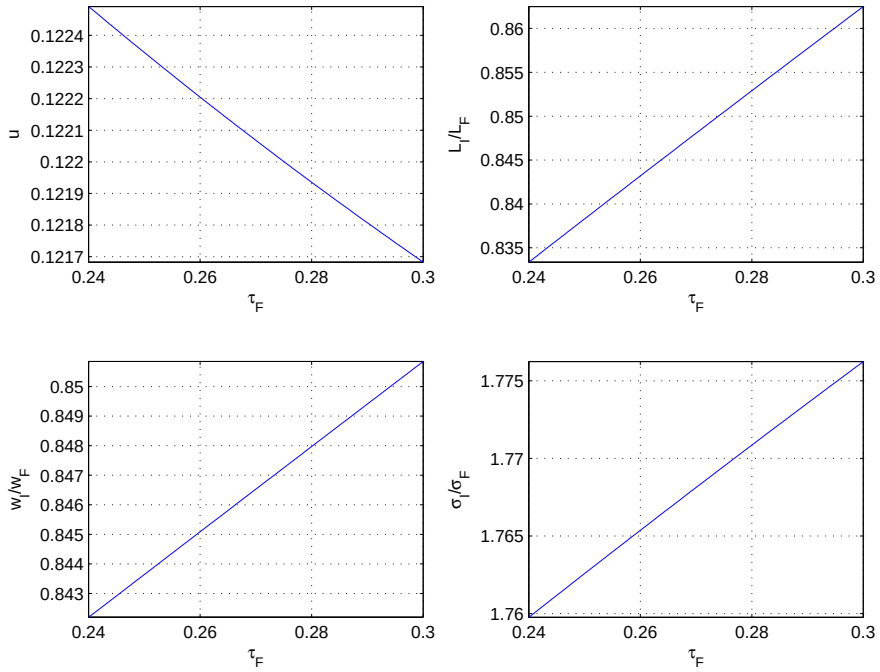

Figure 8: The long-run impact of varying formal sector taxation

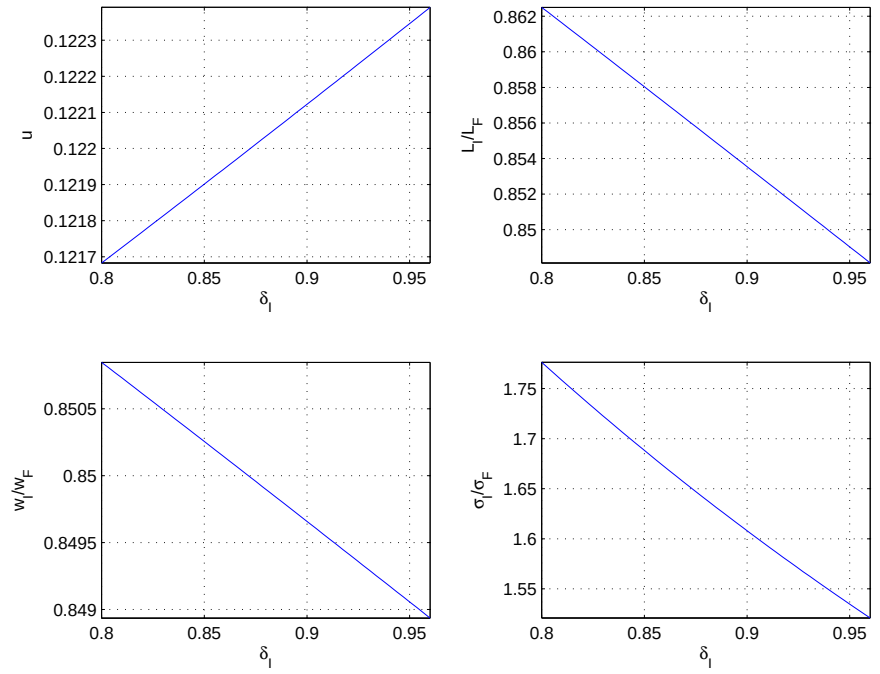

Figure 9: The long-run impact of increasing informal firms' exit rate