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Coordination in the use of money*

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Abstract

Fundamental models of money, while explicit about the frictions that render money socially beneficial, are silent on how agents actually coordinate in its use. This paper studies this coordination problem, providing an endogenous map between the primitives of the environment and the beliefs on the acceptability of money. We show that an increase in the frequency of trade meetings, besides its direct impact on payoffs, facilitates coordination. In particular, for a large enough frequency of trade meetings, agents always coordinate in the use of money. We highlight the underlying properties of money (medium of exchange, record-keeping) that facilitate coordination.

Key Words: Money, Beliefs, Coordination.

JEL Codes: E40, D83

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1 Introduction

The principle that the use of money should be explained by its essentiality is well established among monetary theorists.[1] A precise description of the frictions (e.g., limited commitment, limited record-keeping) that render money necessary if the economy is to achieve socially desirable allocations is a crucial element in models where the use of money is not taken as a primitive of the environment.[2] However, these models take the belief that money is accepted as given, thus assuming away the coordination problem involved in its use. In this paper we step towards filling this gap by exploring how the primitives of the economy and the underlying properties of money impact agents' ability to coordinate in its use.

We cast our analysis in a search model of money along the lines of Kiyotaki and Wright (1993). The key departure from their environment is the assumption that money is not completely fiat. We let the economy experience different states over time and assume that, while money is fiat in a large region, there exist faraway states where money acquires negative intrinsic utility (and it becomes a strictly dominant action not to accept money) and faraway states where money acquires positive intrinsic utility (and it becomes a strictly dominant action to accept money). A precise interpretation of how money may acquire some intrinsic disutility or utility is not important in our analysis, but we think of different states as reflecting changes in the physical characteristics of money or changes in the characteristics of the environment that may impact the effectiveness of money as a medium of exchange. What is important is that the existence of remote states where either accepting or not accepting money is a strictly dominant action prevents the emergence of equilibria where an agent chooses a particular action because he is certain everyone else will always choose the same action in all possible states. This restriction on beliefs allows us to derive a link between the primitives of the economy and agents' beliefs on the acceptability of money.

We first show that if there are enough gains from trade, agents always coordinate in the use of money in the region where it is completely fiat. This result, albeit novel to monetary economies, is analogous to those found in other settings with strategic complementarities where agents cannot coordinate in the same particular set of beliefs in all states of the world. More interestingly, we decompose the impact of an increase in the time discount factor (or an increase in the frequency of trade meetings) on the acceptability of money into two channels: an efficiency channel, which takes the belief that money is accepted by the other agents as given; and a coordination channel, which considers how an increase in the frequency of trade meetings affects the beliefs of an agent about the acceptability of money in states where it has no intrinsic utility. As the frequency of trade meetings increases, the region of parameters where agents coordinate in the use of money expands, and in the limit where the frequency of trade meetings goes to infinity, agents coordinate in the use of money if and only if it is efficient to do so.

[1] For a discussion of the notion of essentiality, see Kocherlakota (1998) and Wallace (2001).

[2] Key references are Kiyotaki and Wright (1989, 1993), Trejos and Wright (1995), Shi (1995, 1997), and Lagos and Wright (2005).

The result that the efficient outcome always emerges when agents become infinitely patient unveils a distinctive feature of the coordination problem involved in the use of money. Two properties of money are particularly important. First, the decision to accept money is not made once but many times. Thus, if an agent exerts effort in order to earn money today, he will be able to spend it sooner, and consequently further opportunities to accept and spend money will also come sooner. The more patient the agent is, the more value he gives to such future opportunities. Second, money is a durable form of record-keeping, i.e., it is evidence that, at some point in the past, no matter how far back it is, an agent produced to someone else, and is thus entitled to current consumption. This ensures that a patient agent will be willing to exert effort in exchange for money even if he believes that it will take a long time until he can spend money, which gives rise to beliefs that money will be widely accepted in the economy.

Our paper relates to the literature on equilibrium selection in dynamic games with complete information, in particular, Frankel and Pauzner (2000) (FP), and Burdzy, Frankel, and Pauzner (2001) (BFP).[3] We share with these papers both the assumption that there exist faraway states where it is strictly dominant to choose a particular action and the result that the ensuing equilibrium is unique.[4] An important difference is that in FP and BFP, an increase in the time discount factor does not help selecting the efficient outcome. In their model, if the time discount factor is large enough, the risk-dominant equilibrium is selected regardless of whether it is efficient. This underpins our argument that, thanks to the underlying properties of money, the coordination problem involved in its use is markedly different from that present in other, non-monetary, settings.

Finally, there is a strand of models within monetary economics that studies how the addition of an intrinsic utility to money may help to reduce the set of equilibria. In overlapping generations models, the focus is on the elimination of monetary equilibria that exhibit inflationary paths (Brock and Scheinkman (1980), Scheinkman (1980)). In search models of money, the objective is to characterize the set of fiat money equilibria that are limits of commodity-money equilibria when the intrinsic utility of money converges to zero (Zhou (2003), Wallace and Zhu (2004), Zhu (2003, 2005)). A result that comes out of this work is that, if goods are perfectly divisible and the marginal utility is large at zero consumption, autarky is not the limit of any commodity money equilibria. This result critically depends on the assumption that there is a sufficiently high probability that the economy reaches a state where fiat money acquires an intrinsic utility. In contrast, our results hold even if the probability that money ever acquires intrinsic utility is arbitrarily small and the probability that it ever acquires intrinsic disutility is equal to one. Most importantly, this literature

[3] It is also related to the literature on equilibrium selection in games with incomplete information, surveyed by Morris and Shin (2003).

[4] One relevant difference is that in a monetary economy, the benefit of exerting effort in exchange for money depends on how agents will behave in the near future and not on how they behave today. This eliminates equilibria in which an agent chooses a particular action in a given period simply because he believes that all the other agents will choose the same action in that period. In order to deal with this problem, FP and BFP assume that each agent has only a small chance of changing his action in any given period in order to prevent the multiplicity of equilibria that arises when agents are allowed to continuously shift from one action to another. We do not need an extra assumption to tackle this issue.

does not explain (it is not meant to) how the fundamentals of the economy and the underlying properties of money affect the ability to coordinate in its use.

The paper is organized as follows. In section 2, we present the model, deliver our main result, and discuss our assumptions. In section 3, we examine how the frequency of trade meetings and the underlying properties of money impact agents' ability to coordinate in its use. In section 4, we conclude. The appendix contains proofs omitted from the main text.

2 Model

2.1 Environment

Our environment is a version of Kiyotaki and Wright (1993). Time is discrete and indexed by t. There are k indivisible and perishable goods and the economy is populated by a unit continuum of agents uniformly distributed across k types. A type i agent derives utility u per unit of consumption of good i and is able to produce one unit of good $i+1$ (modulo k) per period, at a cost $c < u$. Agents maximize expected discounted utility with a discount factor $\beta \in (0,1)$. There is a storable and indivisible object, which we call money. An agent can hold at most one unit of money at a time, and money is initially distributed to a measure m of agents.

Trade is decentralized and there are frictions in the exchange process. There are k distinct sectors, each one specialized in the exchange of one good. In every period, agents choose which sector they want to visit but inside each sector they are randomly and pairwise matched. Each agent faces one meeting per period, and meetings are independent across agents and independent over time. Thus if an agent wants money, he goes to the sector which trades the good he produces and searches for an agent with money. If he has money, he goes to the sector which trades the good he likes and searches for an agent with the good.

In any given period, the economy is in some state $z \in \mathbb{R}$. The economy starts at $z = 0$ and the state changes according to a random process $z_{t+1} = z_t + \Delta z_t$, where Δz_t follows a continuous probability distribution that is independent of z and t, with expected value $\mathrm{E}(\Delta z) \approx 0$ and variance $\mathrm{Var}(\Delta z) > 0$. The state of the economy does not affect the preferences for goods and the production technology. However, it may affect the intrinsic utility of money: there exists a state $\widehat{z} > 0$ and a function $\gamma(z)$ such that (1) if $z < -\widehat{z}$, money provides an intrinsic utility $\gamma(z) < 0$; (2) if $z \in [-\widehat{z}, \widehat{z}]$, money provides no intrinsic utility, i.e., $\gamma(z) = 0$; and (3) if $z > \widehat{z}$, money provides an intrinsic utility $\gamma(z) > 0$.

Throughout, we assume that $\widehat{z}$ is finite but very large, and we are interested in describing how agents behave in states where money provides no intrinsic utility. The probability that states below $-\widehat{z}$ and states above $\widehat{z}$ will ever be reached depends on the stochastic process of Δz. If $\mathrm{E}(\Delta z)$ is zero, both $-\widehat{z}$ and $\widehat{z}$ will eventually be reached with probability one. If, instead, $\mathrm{E}(\Delta z)$ is slightly negative, the probability that $\widehat{z}$ will ever be reached depends on how far $z = 0$ is from $\widehat{z}$. In particular, by choosing a large enough $\widehat{z}$, we can make the probability that money ever acquires a positive intrinsic utility as small as we want. In what follows we show that we can choose $\mathrm{E}(\Delta z)$ to make the probability of reaching either of the faraway states where money starts acquiring some intrinsic utility arbitrarily small, with virtually no effect in any of our results. We will return to this claim in section 2.4.

The Intrinsic Utility of Money The possibility that money may deliver a positive or a negative intrinsic utility is the key dimension in which our model departs from a standard search model of money. We think of the state of the economy as reflecting the physical characteristics of money. In most states money is intrinsically useless. This is captured by the assumption that the intrinsic utility of money is equal to zero if $z \in [-\widehat{z}, \widehat{z}]$, and by the fact that $\widehat{z}$ can be as large as we want, as long as it is finite. However, there are (faraway) states where accepting money entails a cost due to changes in its physical properties. For example, money may become more costly to carry.[5] We capture this idea by assuming that accepting money in a state $z < -\widehat{z}$ gives to the agent a flow utility $\underline{\gamma} < 0$. In turn, there are (faraway) states where changes in the physical properties of money can turn it into a desirable good in and of itself. For example, one can think of the development of a technology that can convert money into a shiner object, say a jewel. We capture this idea by assuming that if an agent holds one unit of money at the beginning of a period in a state $z > \widehat{z}$, he can convert this unit into an intrinsically desirable object, in which case he obtains utility $\overline{\gamma} > 0$.

The only objective of allowing for an intrinsic utility to money is to introduce states in which accepting money is either a strictly dominant or a strictly dominated action. There are many ways in which this can be done. In what follows, we ensure that money is never accepted if $z < -\widehat{z}$ by assuming that $\underline{\gamma} < -\beta u$. Now, in order to ensure that money is always accepted in states $z > \widehat{z}$, we need to consider the expected benefit of holding money at the end of a period under the belief that money will never be accepted by the other agents. In this case, since the benefit of money only comes from the possibility of converting it into an intrinsically desirable object upon reaching some state $z > \widehat{z}$ in a future period, the agent needs to take into account how states evolve over time, as given by the random process Δz_t. If we denote by $\varphi(t)$ the probability that any state $z' > z$ is reached in period $s + t$ and not before, conditional on the agent being in state z in period s, money is accepted in all states $z > \widehat{z}$ if $\sum_{t=1}^{\infty} \beta^t \varphi(t)\overline{\gamma} > c$.[6] Finally, we want to make it clear that the dominant role of money comes from its use as a medium of exchange and not from its intrinsic utility, i.e., even if money may have some intrinsic utility, the agent still prefers to use it as a medium of exchange if he believes that other agents will do the same. For this reason, we further assume that $\overline{\gamma}[1 - m \sum_{t=1}^{\infty} \beta^t \varphi(t)] < (1 - m)u$.[7] In section 2.4 we argue that a more general

[5] Alternatively, accepting money may become costly due to changes in the environment that increase the transaction costs of using money.

[6] An agent that accepts money at $\widehat{z}$ will get $\beta^t \bar{\gamma}$ after t periods with probability $\varphi(t)$. An agent at $z > \widehat{z}$ will on average get $\bar{\gamma}$ sooner, hence his incentives for accepting money are larger.

[7] An agent that converts his unit of money in a "jewel" gets $\bar{\gamma}$ and leaves the game. A lower bound for the payoff of

specification of the function $\gamma(z)$ would not affect the substantive results of the paper.

2.2 Benchmark case

We initially consider the benchmark case where $\widehat{z} = \infty$ and money is completely fiat. We are interested in the behavior of an agent without money, who is asked to produce in exchange for money. The decision of an agent on whether to offer money in exchange for a desirable good is trivial in our model, since the most an agent can obtain with one unit of money is one unit of good, and all desirable goods provide the same utility u.

In principle, the behavior of an agent may depend on the history of states and, consequently, on the agent's belief about the behavior of all other agents in all future periods, and after every possible history of states. A natural approach though if money is completely fiat is to look at equilibria where agents' behavior do not depend on the history of states. One such equilibrium is autarky, where each agent believes all other agents will never accept money. Under some conditions on parameters, another equilibrium is money, in which each agent believes all other agents will always accept money.

Let $V_{1,z}$ be the value function of an agent with money in state z if he believes all other agents will always accept money, and let $V_{0,z}$ be defined in a similar way for an agent without money. We have

$$V_{1,z} = m\beta E_z V_1 + (1-m)\left(u + \beta E_z V_0\right),$$

and

$$V_{0,z} = m\left[\sigma\left(-c + \beta E_z V_1\right) + (1-\sigma)\,\beta V_0\right] + (1-m)\beta E_z V_0,$$

where $\beta E_z V_1$ is the expected payoff of carrying one unit of money into the next period, $\beta E_z V_0$ is the expected payoff of carrying zero units of money into the next period, and $\sigma \in [0,1]$ is the probability that the agent accepts money. Consider the expression for $V_{1,z}$. An agent with money goes to the sector that trades the good he likes. In this sector, there is a probability m that he meets another agent with money and no trade happens. There is also a probability $(1-m)$ that he meets an agent without money, in which case they trade, the agent obtains utility u, and moves to the next period without money. A similar reasoning holds for an agent without money.

Assume that $\sigma = 1$. This implies that, for all z,

$$V_{1,z} - V_{0,z} = (1-m)\,u + mc.$$

It is indeed optimal to always accept money as long as $-c + \beta V_{1,z} \geq \beta V_{0,z}$, i.e.,

$$\beta\left[(1-m)\,u + mc\right] \geq c. \tag{1}$$

an agent that decides to take money to the market is given by the probability of meeting someone who produces the good he likes times the benefit of getting the good, $(1-m)\,u$, plus the probability of meeting another buyer instead of a seller (m) times the value of starting next period with money. A lower bound for the value of money in this case is given by $\sum_{t=1}^{\infty} \beta^t \varphi(t)\overline{\gamma}$, since the agent can convert money in a "jewel" whenever he is in a state $z > \widehat{z}$, and as discussed in the previous footnote, $\sum_{t=1}^{\infty} \beta^t \varphi(t)\overline{\gamma}$ is a lower bound for the expected value of money in this case.

Thus, if money is completely fiat and (1) holds, there always exist an autarkic equilibrium and a monetary equilibrium.[8]

2.3 General case

We now consider the case where $\widehat{z}$ is finite, so there are states where money carries some intrinsic utility. We are interested in the behavior of agents without money who are asked to produce in exchange for money in states where money yields no intrinsic utility. In what follows, we focus on equilibria in cut-off strategies, i.e., strategies where an agent accepts money if and only if the current state is above some threshold state z^*, where $z^* \in [-\infty, \infty]$. To be clear, we do not impose that agents have to follow a cut-off strategy, we allow for deviations where an agent chooses a strategy that is not of a cut-off type. Proposition 1 summarizes our main result.

Proposition 1 *Let $\widehat{z} \in \mathbb{R}$ and*

$$\sum\nolimits_{t=1}^{\infty} \beta^t \varphi(t) \left[(1-m)\, u + mc\right] > c. \tag{2}$$

Then, there exists a unique equilibrium in cut-off strategies. In this equilibrium, money is accepted if and only if $z \geq -\widehat{z}$.

Proposition 1 shows that, unless accepting money implies an intrinsic disutility, agents will always coordinate in the use of money if (2) holds.[9] The proof of Proposition 1 is organized in two steps. First, taking beliefs about the behavior of others as given, we study the decision of an agent and the relevant value functions. We then characterize the unique equilibrium in cut-off strategies.

We start by showing that if an agent believes that all the others are following a cut-off strategy at $z \in [-\widehat{z}, \widehat{z}]$, then the best response is also a cut-off strategy. This best response is unique and we denote the corresponding cut-off state by $Z(z)$. This is proved in Lemma 1.

Lemma 1 *For every $z^* \in [-\widehat{z}, \widehat{z}]$, there exists a unique $Z(z^*) \in [-\widehat{z}, \widehat{z}]$ such that, if an agent believes that all the other agents follow a cut-off strategy at z^*, then he produces in exchange for money if and only if $z \geq Z(z^*)$.*

Proof. See the Appendix. ■

The decision about accepting money is actually a decision between an action and an option. The agent can accept money now, which entails a cost c but gives him money that can be used to buy the good he likes next period. Alternatively, the agent can wait for another opportunity. The

[8] Kiyotaki and Wright (1993) show that there exists an equilibrium where agents are indifferent between accepting and not accepting money. In this (unstable) mixed strategy equilibrium, the probability of accepting money is increasing in c and decreasing in u. A similar equilibrium also exists here.

[9] The case where (2) holds with equality is uninteresting as it only occurs in a set of parameters with measure zero. We discuss the case where the reverse of (2) holds in section 2.4.

value of accepting money now and the value of the option of accepting money in the next period are both increasing in the likelihood that money will be accepted later.

By accepting money in the current period, an agent puts himself in the position of getting a reward at the first time the economy reaches a state where money is accepted by others. By delaying the moment of accepting money, the agent saves in effort cost, because the present value of future effort cost is lower than c, but the agent might miss on an opportunity to spend money quickly. If an agent believes that all other agents are following a cut-off strategy at some state z^*, and z is much larger than z^*, it makes sense to accept money now, as the probability of a reward in the following period is relatively high. In contrast, if z is much smaller than z^*, it makes more sense to wait for a future opportunity of accepting money, since there will probably be other opportunities of accepting money before the agent can spend it. Lemma 1 shows that this intuition applies more generally: if others are following a cut-off strategy at some state z^*, the payoff of accepting money for an agent is increasing in z. Hence, the best response to cut-off strategies is also a cut-off strategy.

Now let $V_1(z_1, z_2, z_3)$ be the payoff of having money at the end of the period if the current state is z_1, all the other agents are following a cut-off strategy at z_2, and the agent is following a cut-off strategy at z_3 and let $V_0(z_1, z_2, z_3)$ be the payoff of having no money in the same situation. Then the net expected payoff of producing in exchange for money is given by

$$-c + \beta V_1(z_1, z_2, z_3) - \beta V_0(z_1, z_2, z_3).$$

Lemma 2 shows that if an agent is following a cut-off strategy at $z \in [-\widehat{z}, \widehat{z}]$ and all other agents are following a cut-off strategy at $z' \in [-\widehat{z}, \widehat{z}]$, then the net expected payoff of producing in exchange for money in state z is strictly decreasing in z'.

Lemma 2 *For any z and z' in $[-\widehat{z}, \widehat{z}]$, the net expected payoff of producing in exchange for money $-c + \beta V_1(z, z', z) - \beta V_0(z, z', z)$ is strictly decreasing in z'.*

Proof. See the Appendix. ■

The intuition for Lemma 2 is similar to the intuition for Lemma 1: as the threshold for accepting money z' increases, the likelihood that money will be accepted by others in the near future decreases, hence incentives for delaying the moment of producing in exchange for money increase.

Lemma 3 then uses Lemmas 1 and 2 to show that if an agent believes all others are following a cut-off strategy at z, then his expected payoff employing the same cut-off strategy is sufficient to determine the optimal behavior in state z.

Lemma 3 *For any $z \in [-\widehat{z}, \widehat{z}]$, if*

$$\beta \left[V_1(z, z, z) - V_0(z, z, z) \right] > c, \tag{3}$$

it is optimal to accept money in state z. If the inequality is reversed, then it is optimal not to accept money in state z.

Proof. See the Appendix. ■

The key element in the proof of Lemma 3 is the fact that, since the random process Δz_t follows a continuous probability distribution that is independent of the current state z, it must be that for all z and z' in $[-\widehat{z}, \widehat{z}]$,

$$V_1(z, z, z) - V_0(z, z, z) = V_1(z', z', z') - V_0(z', z', z').$$

In words, once everyone is following the same cut-off strategy at some state in $[-\widehat{z}, \widehat{z}]$, payoffs do not depend on how far the cut-off state is from the dominant regions. Thus a corollary of Lemma 3 is that if (3) holds, there exists an equilibrium in which all agents follow a cut-off strategy at $z = -\widehat{z}$: the assumption that $\gamma(z) < -\beta u$ for all $z < -\widehat{z}$ implies it is strictly optimal not to accept money in all states $z < -\widehat{z}$, and (3) implies it is strictly optimal to accept money in all states $z \geq \widehat{z}$ if agents are following a cut-off strategy around $\widehat{z}$. A similar reasoning implies that, if the reverse of (3) holds, there exists an equilibrium in which all agents follow a cut-off strategy at $z = \widehat{z}$.

Moreover, there can be no symmetric equilibrium in which agents follow a cut-off strategy at some state $z \in (-\widehat{z}, \widehat{z})$. If (3) holds and all other agents are following a cut-off strategy at some $z \in (-\widehat{z}, \widehat{z})$, then Lemma 2 implies that the agent's best response is to follow a cut-off strategy at some $z' < z$. Similarly, if the reverse of (3) holds and all the other agents are following a cut-off strategy at some $z \in (-\widehat{z}, \widehat{z})$, then Lemma 2 implies that the agent's best response is to follow a cut-off strategy at $z' > z$.

Summing up, if one restricts attention to symmetric cut-off strategies, there exists a unique equilibrium. How about equilibria in which agents follow distinct cut-off strategies? Lemma 4 proves that such equilibria cannot exist.

Lemma 4 *There are no equilibria where agents follow different cut-off strategies.*

Proof. See the Appendix. ■

The intuition for Lemma 4 is that agents following different cut-off strategies would have different payoffs: one agent would be accepting money even around states where others do not accept it, while another one would only accept money when surrounded by states where money is accepted by many others, but they cannot both be indifferent between accepting and not accepting money.

Hence in order to characterize behavior in the unique equilibrium in cut-off strategies as a function of the primitives, it is sufficient to consider payoffs induced by a symmetric strategy profile where all agents choose the same cut-off z. Consider then the problem of an agent in state z in period s. If he follows a cut-off strategy at z and believes all others follow the same cut-off strategy, the net expected payoff of holding one unit of money at the end of period s, $V_1(z,z,z) - V_0(z,z,z)$, is given by

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \left[\int_z^{\infty} \left(V_{1,z'} - V_{0,z'} \right) dF(z'|t) \right]. \tag{4}$$

In words, conditional on the belief that money is only accepted in states $z' > z$, the net expected payoff of holding money is given by the discounted probability that the first time the economy reaches some state $z' > z$ after period s happens in period $s+t$ (which is given by $\beta^t \varphi(t)$) multiplied by the net expected value of having money in state z', that is, the expected value of $V_{1,z'} - V_{0,z'}$. The cdf of z' is given by $F(z'|t)$, i.e., the probability that the state of the economy is below or equal to z' conditional on the event that the first time the economy reaches some state $z' > z$ after period s happens in period $s + t$.

Now, if an agent follows a cut-off strategy at z and if he believes that all the other agents follow the same cut-off strategy, the value of having money in some state $z' > z$ is given by

$$V_{1,z'} = m\beta E_{z'} V_1 + (1-m) \left(u + \beta E_{z'} V_0 \right), \tag{5}$$

while the value of not having money in some state $z' > z$ is

$$V_{0,z'} = m \left(-c + \beta E_{z'} V_1 \right) + (1-m) \beta E_{z'} V_0. \tag{6}$$

The expected values $E_{z'} V_0$ and $E_{z'} V_1$ are complicated objects, but the analysis is vastly simplified because for any $z' > z$

$$V_{1,z'} - V_{0,z'} = (1-m)u + mc. \tag{7}$$

We can then substitute (7) in (4) and rewrite (3) as

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \left[(1-m) \, u + mc \right] > c.$$

which is the condition in (2).

2.4 Discussion

The condition in (2) holds for any $\widehat{z} \in \mathbb{R}$. The role of the regions $z > \widehat{z}$ and $z < -\widehat{z}$ is simply to introduce states where accepting money is either a strictly dominant or a strictly dominated action. In particular, $\widehat{z}$ can be as large as we want, and it is in this sense that we think of the states where money acquires a negative or a positive intrinsic utility as being faraway states.

An implication of (2) is that coordination in the use of money in states where money is completely fiat does not depend on the actual values of $\underline{\gamma}$ and $\overline{\gamma}$ in the faraway states, irrespective of the distance between the current state and the frontier states $-\widehat{z}$ and $\widehat{z}$. This result hinges on the way the intrinsic utility of money was introduced in our model. The value of $\underline{\gamma}$ does not matter even if the agent is at $-\widehat{z}$ because an agent only faces some disutility if he accepts money. Since money is not accepted in states $z < -\widehat{z}$, no agent actually incurs such disutility on the equilibrium path. In turn, the value of $\overline{\gamma}$ does not matter even if the agent is at $\widehat{z}$ due to the assumption that $\overline{\gamma}[1 - m \sum_{t=1}^{\infty} \beta^t \varphi(t)] < (1 - m)u$: if the agent is certain that money will be accepted, he prefers using it as a medium of exchange instead of keeping it to enjoy its intrinsic utility. This assumption is appealing as it makes it transparent that, from the perspective of an agent, the key force driving the coordination in the use of money is the belief on its acceptability as a medium of exchange, and not the fact that it may acquire an intrinsic utility. It is intuitive that the incentives to accept money and thus the coordination in its use would only strengthen if we increased the intrinsic utility of money by assuming that $\overline{\gamma}[1 - m \sum_{t=1}^{\infty} \beta^t \varphi(t)] \geq (1 - m)u$.

Proposition 1 characterizes equilibrium in the region of parameters where (2) holds. What if the reverse of (2) holds but money is still an equilibrium in the benchmark model? We did not consider this possibility in Proposition 1 because the assumptions $\sum_{t=1}^{\infty} \beta^t \varphi(t)\overline{\gamma} > c$ and $\overline{\gamma}[1 - m \sum_{t=1}^{\infty} \beta^t \varphi(t)] < (1 - m)u$ cannot be both satisfied if the reverse of (2) holds.[10] The problem in relaxing the latter assumption is that, in states $z > \widehat{z}$, the intrinsic utility of money starts to dominate its role as a medium of exchange. As a result, an agent may want to hoard money in states that are close enough to $\widehat{z}$ in which money is yet completely fiat. The simplest way to avoid this complication and prevent the intrinsic utility of money in the dominant regions to affect behavior in other states is to assume that when $z > \widehat{z}$, agents are forced to accept money by some special agent in the economy. Under this assumption, Lemma 3 implies that if the reverse of (2) holds, then agents do not accept money in all states $z \leq \widehat{z}$ in the unique equilibrium in cut-off strategies.[11]

The assumption that Δz follows a random walk makes the problem identical at every state z. While this is an important assumption, small perturbations on the process for Δz would not significantly affect the equilibrium conditions. Starting from a given state z, if $\mathrm{E}(\Delta z)$ is slightly negative (positive), the probability of reaching the region where accepting money is a strictly dominant (dominated) action can be made arbitrarily small by choosing a large enough $\widehat{z}$. Such long term probabilities are not important in the computation of (2). The fact that money will eventually acquire some intrinsic utility does not affect the condition in (2), which determines agents' behavior in $[-\widehat{z}, \widehat{z}]$. What is important is the set of probabilities $\varphi(t)$ of reaching nearby states where money is accepted as a medium of exchange. Now, the set of probabilities $\varphi(t)$ are very similar if $\mathrm{E}(\Delta z) = 0$ or if $\mathrm{E}(\Delta z) = \eta$, where η is a very small negative number. Thus we can

[10] The assumption that guarantees money is accepted for $z > \widehat{z}$ provides a lower bound for $\bar{\gamma}$, while the assumption that makes sure money is spent yields an upper bound for $\bar{\gamma}$. Combining both leads to a condition equivalent to (2). Hence as long as (2) holds, we can always find a value of $\bar{\gamma}$ that satisfies the conditions on parameters, but that is never true if (2) doesn't hold.

[11] This special agent could be interpreted as a government and the region $z > \widehat{z}$ would correspond to states where the government enforces the use of money or accepts it as payment for taxes. We chose not to follow this route because it does not offer a clear link between the elements of our model and the region in which accepting money is a strictly dominant action.

make the probability that money will have a negative intrinsic utility in the long run arbitrarily close to one, without significantly affecting the condition for money being accepted in all states where it has no intrinsic negative utility.[12]

The assumption that Δz follows a random walk implies that, in the long run, the economy will usually be at states outside the $[-\widehat{z}, \widehat{z}]$ interval. However, a small modification of the random walk process could rule out this outcome without significantly affecting our results. Consider a process such that $\mathrm{E}(\Delta z) = -\eta$ for any $z > 0$ and $\mathrm{E}(\Delta z) = \eta$ for any $z < 0$. For η sufficiently small, the set of probabilities of reaching a nearby state in the following periods would not be substantially affected, and thus the condition for a unique monetary equilibrium would be very similar to (2). We can then make sure that the economy will rarely be outside of the $[-\widehat{z}, \widehat{z}]$ interval by choosing a large enough $\widehat{z}$.

We have considered the case of a continuous state space but our results can be extended to a discrete state space, i.e., the case where the set of states is given by $\mathbb{Z}$. We present this case in the Appendix. The key difference is that, in the discrete case, if the economy is in some state z in the current period, there is a positive probability that it will be in the same state after a couple of periods. This gives rise to an intermediate region of parameters were multiple equilibria exist. Intuitively, the possibility of the economy being exactly at the same point in the future allows agents to coordinate on arbitrary beliefs. However, except for this multiplicity region, results are analogous to the continuous case. Moreover, as the frequency of trade meetings go to infinity, the region of multiple equilibria disappears.

We considered a particular form for the function $\gamma(z)$, which is discontinuous at $z = -\widehat{z}$ and at $z = \widehat{z}$. An arguably more natural assumption would be a smooth function $\gamma(z)$ which is negative, continuous and strictly increasing up to $z = -\widehat{z}$, is equal to zero for all $z \in [-\widehat{z}, \widehat{z}]$, and is positive, continuous, and strictly increasing from $\widehat{z}$ on. We did not choose an specification along these lines because it is less tractable in the region of states where money starts acquiring some intrinsic disutility or utility but it generates exactly the same results in the region of states where money is completely fiat. To see this, simply note that the 4 lemmas proved above do not depend on how we specify the intrinsic utility of money in states outside the interval $[-\widehat{z}, \widehat{z}]$.

Finally, note that all 4 lemmas hold in the case where $\widehat{z} = \infty$ and money is completely fiat. Thus, a corollary of our analysis is that if $\widehat{z} = \infty$, there is generally no equilibrium where agents follow a cut-off strategy at some state $z \in \mathbb{R}$. If all agents are following a cut-off strategy at z and (2) holds, Lemma 3 implies that the best response is a cut-off strategy at $Z(z) < z$. In turn, if the reverse of (2) holds, Lemma 3 implies that the best response is a cut-off strategy at $Z(z) > z$. Thus, autarky (which could be seen as agents following a cut-off rule at $z = \infty$) and money (a cut-off rule at $z = -\infty$) are the unique equilibria if money is completely fiat. An implication of

[12] This highlights the contribution of this paper: we are not constructing an equilibrium where money is valuable by working backwards from some distant future where money acquires positive intrinsic utility with a sufficiently high probability. The contribution lies in unveiling how the primitives of the economy, by changing the left-hand side of (2), impact agents' ability to coordinate in the use of money.

this result is that the benchmark case reproduces exactly the same set of pure-strategy equilibria present in a standard search model of money along the lines of Kiyotaki and Wright (1993).

3 Coordination

In this section we first look at how changes in the time discount factor impact agents' ability to coordinate in the use of money. We then consider how the underlying properties of money help to explain how agents coordinate in its use.

3.1 Coordination and the frequency of trade meetings

A distinguishable result of our model concerns the effects of the time discount factor on the coordination in the use of money. In what follows, we look at these effects in detail. Note that an increase in the time discount factor can be interpreted as a reduction in the time interval between two consecutive periods. In our environment, since there is one meeting per period, this amounts to an increase in the arrival rate of trading opportunities.[13] Thus, in the discussion below, we think of an increase in the time discount factor as capturing an increase in the frequency of trade meetings.

To better understand the role of β, it is useful to rewrite (2) as

$$\lambda\beta\left[(1-m)\,u+mc\right]>c, \tag{8}$$

where

$$\lambda=\sum_{t=1}^{\infty}\beta^{t-1}\varphi\left(t\right). \tag{9}$$

We can decompose the overall impact of an increase in β on the existence of the monetary equilibrium into two separate effects: an efficiency effect and a coordination effect. The efficiency effect takes as given the belief that money is accepted by all the other agents, and considers how an increase in β affects the incentives of an agent to produce in exchange for money. The intuition underlying this effect is straightforward: since money acquired today only commands value in the future, an increase in β implies a larger gain from trade, thus increasing the incentives of a single agent to accept money.

The efficiency effect is not novel and is present in any model of money that separates between the act of producing in exchange for money and the act of consuming with money previously acquired. However, since models of money take as given the belief that money is accepted as a medium of

[13] Alternatively, we could consider a setting in which β is fixed, but agents are randomly and anonymously matched in pairs $n\geq 1$ times in each period. An increase in n would then amount to an increase in the arrival rate of trading opportunities. We obtain the same results with this alternative specification, but since the agent's decision on whether to accept money depends on the state of the economy, when there is more than one meeting in a period, we need to assume that the state of the economy changes across meetings and not across periods. This way, if β is the period discount factor and there are n meetings in a period, then $\beta^{\frac{1}{n}}$ is the discount factor in between meetings. The analysis then is exactly the same as in section 2.3, with $\beta^{\frac{1}{n}}$ replacing β. As n goes to ∞, this discount factor goes to one.

exchange, they do not have much to say about the coordination problem involved in the use of money. This is where a key contribution of this paper lies. In addition to the efficiency channel, we uncover a second channel through which an increase in β impacts the existence of a monetary equilibrium. This coordination channel considers how an increase in β affects the beliefs of agents about the acceptability of money in states where money has no intrinsic utility.

In order to better understand the coordination effect, remember that the left-hand side of (8) corresponds to the net benefit of having money at the end of the period in some state $z \in [-\widehat{z}, \widehat{z}]$, if the agent holds the belief that money is accepted if and only if $z' \geq z$. In proposition 1 , we have shown that if the net benefit of holding money under the cut-off belief is larger than the production cost c, then money is accepted in all states where it has no intrinsic utility. Since the value of λ in (9) strictly increases with β, a larger β facilitates the coordination in the use of money. The intuition runs as follows. If an agent expects to meet many partners in a short period of time, he is willing to accept money even if he believes that a large number of his future partners will not accept money. That affects beliefs of all other agents: they know others will be willing to produce in exchange for money, even if they hold relatively pessimistic beliefs. Consequently, pessimistic beliefs cannot be an equilibrium.

It is instructive to think of an economy where one agent has a different and, say, low time discount factor – call it $\beta_L < \beta$. Whether money will be accepted in $[-\widehat{z}, \widehat{z}]$ depends on whether (2) holds: it is the other agents's β that affects the coordination channel. If (2) holds, the impacient agent will also accept money in states far enough from either dominant region as long as his impatience is not large enough to eliminate his gains from trade (i.e., as long as $\beta_L [(1-m) u + mc] > c$).

We can isolate the efficiency effect by fixing λ and looking at how changes in β affect the left-hand side of (8). In turn, we can isolate the coordination effect by fixing β in (8) and looking at how changes in β affect the value of λ in (9), and how changes in λ affect the left-hand side of (8). We obtain

$$\frac{\partial \lambda \beta}{\partial \beta} = \left\{ \underbrace{\sum_{t=1}^{\infty} \beta^{t-1} \varphi(t)}_{\text{efficiency effect}} + \underbrace{\sum_{t=1}^{\infty} (t-1) \beta^{t-1} \varphi(t)}_{\text{coordination effect}} \right\}.$$

Both the efficiency and the coordination effects are positive and increasing in β. Moreover, as β increases, the role of the coordination effect becomes relatively more important in explaining the existence of the monetary equilibrium.

It is particularly interesting to look at the extreme case where β converges to one. To do so, consider first the scenario where the stochastic process that governs Δz is symmetric, that is, $\mathrm{E}(\Delta z) = 0$. In this case, since the probability that the economy will eventually reach a state to the right of the current state is equal to one (that is, $\sum_{t=1}^{\infty} \phi(t) = 1$) it must be that λ converges to one as β converges to one. This means that if one restricts attention to the region of parameters where money is an equilibrium in the benchmark model of section 2.2, then if $\beta = 1$, money is always the

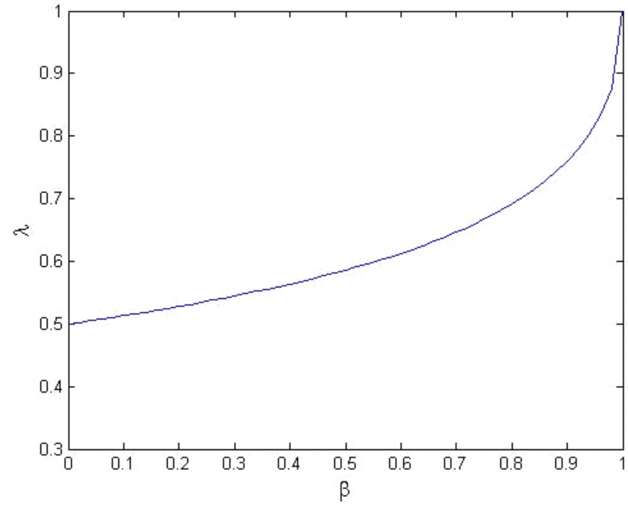

Figure 1: Normal case

unique equilibrium in the model of section 2.3.

It is possible to construct an example where the probability of ever getting to the region where holding money is a strictly dominant action is arbitrarily small and still, as β goes to 1, λ converges to one. Let $\mathrm{E}(\Delta z) = \eta$ for some $\eta < 0$. Since Δz_t follows a continuous probability distribution, as η approaches zero and β goes to one, λ converges to one. Now, for any $\eta < 0$ there exists a large enough $\widehat{z}$ so that the probability of ever reaching states $z > \widehat{z}$ is arbitrarily small, and the probability of reaching states $z < -\widehat{z}$ is equal to one. Therefore, as β goes to one, money is the unique equilibrium in the general model in the region where it is an equilibrium in the benchmark model, despite the probability that money will ever acquire a positive intrinsic utility being arbitrarily small and the probability that it will eventually acquire a negative intrinsic utility being equal to one. A larger β helps agents to coordinate in the monetary equilibrium even if $\mathrm{E}(\Delta z) < 0$ and the economy is bound to reach a region where accepting money is a strictly dominated action.

An inspection of the expression in (9) shows that λ is increasing and convex in β. In the case $\mathrm{E}(\Delta z) = 0$, we also observe that $\lambda = 1/2$ as β approaches 0. As an example, Figure 1 shows how λ varies with β for a normal process assuming $\mathrm{E}(\Delta z) = 0$. The probabilities $\varphi(t)$ are obtained from Monte Carlo simulations (they do not depend on the variance $\mathrm{Var}(z)$).

The conditions for existence of the monetary equilibrium in the benchmark model and in the general model depend on β, m, u and c. Normalizing $c = 1$ and assuming $m = 1/2$, which maximizes the number of trade meetings, the possible equilibria are drawn in Figure 2. Above the solid curve, both money and autarky are equilibria in the benchmark model. Above the dotted line, money is accepted in all states $z \in [-\widehat{z}, \widehat{z}]$ in the unique equilibrium of the general model. The distance

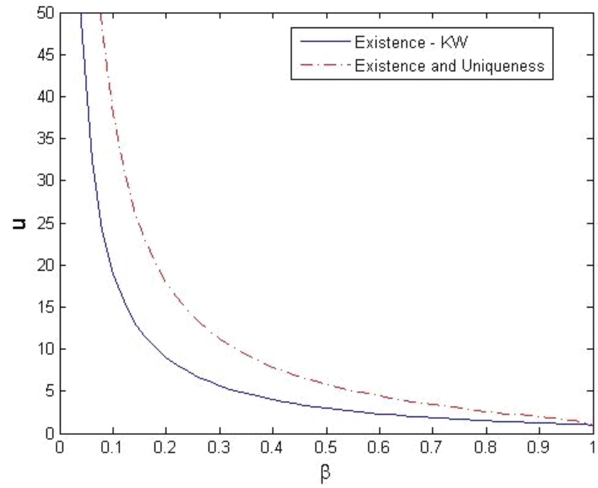

Figure 2: Equilibrium conditions in (1) and (2)

between both lines decreases with β and vanishes as β goes to 1.

3.2 Coordination and the properties of money

The properties of money that explain its societal benefits also help explain the relative ease with which agents coordinate in its use. Two properties, which we discuss below, are particularly revealing.

First, it is an established idea that money achieves desirable allocations because it works as a form of memory, i.e., as a record-keeping of past actions. Intuitively, by constituting hard evidence that an agent has produced at some point in the past, money is a claim to consumption in the present. Importantly, this claim is long lasting in the sense that it remains valid irrespective of when the past production that generated it took place. Kocherlakota (1998) builds on this idea to show that, in a large class of environments, if one wants to design a soft technology that replicates the allocations achieved with money, this technology must include the past actions of an agent in all previous periods.[14] Kocherlakota (1998) though takes as given the belief that money is accepted by all agents, and thus abstracts away from the coordination problem involved in its use.

In order to understand why the role of money as evidence of past production also matters for coordination, consider condition (2) in Proposition 1, which gives the region of parameters where agents coordinate in the use of money. Remember that the left-hand side of (2) gives the net

[14] There are environments in which less memory is required if one wants to replicate monetary allocations. For instance, consider a modified version of our environment in which $m = 1/2$ and, in each sector of the decentralized market, an agent with money always meets an agent without money. In this environment, if one wants to replace money with a record-keeping technology, it is enough to have a technology that only keeps track of the agent's action in the previous period. This is so because exchange happens in every meeting between a consumer and a producer, so observing that an agent has produced in the previous period is equivalent to observing that an agent holds one unit of money in the current period.

expected payoff of producing in exchange for money under the belief that all the other agents follow a cut-off strategy at the current state of the economy. This region implicitly assumes that money perfectly records the fact that, at some point in the past, an agent produced to another agent who liked his good. This is so because it may take many periods until an agent reaches a state at which money is accepted, i.e., a state to the right of the current state of the economy. Thus if one wants to design a record-keeping technology that includes all the information that allows agents to coordinate in the use of money in the region given by (2), then this technology must include the past actions of an agent in all previous periods. Summing up, the role of money as memory not only helps it achieve desirable allocations, as shown by Kocherlakota (1998), but also helps agents to coordinate in its use.[15]

The second property of money that matters for coordination is its role as a medium of exchange. To understand why this is important, it is instructive to draw a parallel between the coordination problem involved in the use of money, and the coordination problem present in other settings. First, an important element of a monetary economy is the fact that the benefit of exerting effort in exchange for money depends not on whether agents accept money today, but on whether agents accept money in the future. This prevents the sort of multiplicity of equilibria that arises in settings where the decision to exert effort depends on the current decisions of the other agents. Second, and more importantly, the decision to accept money is not made once but many times, thus the sooner the agent accepts and uses money, the sooner he will be able to accept and use it again. To illustrate this effect, consider a modified version of the environment in section 2.1, in which an agent leaves the economy after spending money for the first time. In order to keep the environment stationary, assume that, after leaving the economy, the agent is replaced by a new agent with zero units of money. This implies that

$$V_{1,z'} = m\beta E_{z'} V_1 + (1-m)u,$$

while $V_{0,z'}$ is as in (6). Thus, we obtain

$$V_{1,z'} - V_{0,z'} = (1-m)\,u + mc - (1-m)\beta E_{z'} V_0.$$

As compared to the case where money can be accepted and spent many times, $V_{1,z'} - V_{0,z'}$ is now reduced by the term $(1-m)\beta E_{z'} V_0$. This implies that the incentives of an agent to produce in exchange for money are weaker when money can only be accepted and spent once, and especially so if β is large. In particular, due to the presence of $(1-m)\beta E_{z'} V_0$ in the expression above, it is not the case anymore that, as the discount factor goes to one, the condition for the existence of the monetary equilibrium becomes $u > c$, so that agents always coordinate in the use of money. Thus, the fact that the agent can accept and use money many times helps to coordinate in its use.

[15] What is important for coordination is not only that an agent meets a producer of the good he likes, but also that the producer is willing to accept money. Hence even though much less memory may be required in some settings to replicate the benefits of money (see footnote 14 for an example), infinite memory is always required to ensure that agents coordinate in the use of money in the region given by (2).

4 Final remarks

The notion of essentiality, by emphasizing that the societal benefits of money come from the fact that it achieves desirable allocations, helps identify models in which the use of money is justified on grounds of efficiency (Wallace (2001)). However, it does not say much about how agents actually end up coordinating in the use of money, and how do the primitives of the environment and the underlying properties of money impact such coordination. We think of our paper as an initial step in this direction.

We have chosen to present our analysis in a search model of money along the lines of Kiyotaki and Wright (1993), a choice that is mainly driven by tractability reasons. This choice though comes at a cost as their environment is special in some dimensions, particularly the indivisibility of money and the indivisibility of goods – assumptions that have been relaxed in subsequent papers such as Trejos and Wright (1995), Shi (1995, 1997) and Lagos and Wright (2005). We leave the extension to these settings for future work. Since the environment in Kiyotaki and Wright (1993) combines key elements that matter for the problem of coordinating in the use of money in a tractable manner, we see it as a natural starting point.

Our results highlight that an important primitive that impacts the coordination in the use of money is the rate at which trade meetings take place. In particular, as long as exchange is efficient, i.e., the utility of consumption is larger then the cost of production, agents always coordinate in the use of money if the frequency of trade meetings is large enough. This is so even if the probability that money will eventually acquire a positive intrinsic utility is arbitrarily small and the probability that it will eventually acquire a negative intrinsic utility is equal to one. Even though the existence of dominant regions is necessary to rule out equilibria that are insensitive to fundamentals, the result that agents are likely to coordinate in the use of money is not driven by expectations of enjoying its intrinsic utility in some states. The underlying properties of money, particularly the fact that agents accept and use it many times, and the fact that money is a resilient record-keeping technology, help explain the relative ease with which agents coordinate in its use, as compared to other settings where coordination matters.

References

[1] Brock, W. and Scheinkman, J., 1980, Some Remarks on Monetary Policy in an Overlapping Generations Model, in Models of Monetary Economies, edited by John Kareken and Neil Wallace, *Federal Reserve Bank of Minneapolis*, Minneapolis.

[2] Burdzy, K., Frankel, D., and Pauzner A., 2001, Fast Equilibrium Selection by Rational Players Living in a Changing World, *Econometrica* 68, 163-190.

[3] Frankel, D. and Pauzner, A., 2000, Resolving indeterminacy in dynamic settings: the role of shocks, *Quarterly Journal of Economics* 115, 283-304.

[4] Kiyotaki, N. and R. Wright, 1989, On Money as a Medium of Exchange, *Journal of Political Economy* 97, 927-954.

[5] Kiyotaki, N. and R. Wright, 1993, A Search-Theoretic Approach to Monetary Economics, *American Economic Review* 83, 63-77.

[6] Kocherlakota, N., 1998, Money is Memory, *Journal of Economic Theory*, 81, 232-51.

[7] Lagos, R. and Wright, R., 2005, A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy* 113, 463-484.

[8] Morris, S. and Shin, H., 2003, Global games: theory and applications, in Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society), edited by M. Dewatripont, L. Hansen and S. Turnovsky; Cambridge University Press.

[9] Scheinkman, J., 1980, Discussion, in Models of Monetary Economies, edited by John Kareken and Neil Wallace, *Federal Reserve Bank of Minneapolis*, Minneapolis.

[10] Shi, S., 1995, Money and Prices: A Model of Search and Bargaining, *Journal of Economic Theory*, 67, 467-496.

[11] Shi, S., 1997, A Divisible Search Model of Fiat Money, *Econometrica* 65, 75-102.

[12] Trejos, A. Wright, R., 1995, Search, Bargaining, Money, and Prices, *Journal of Political Economy* 103, 118-141.

[13] Wallace, N., 2001, Whither Monetary Economics?, *International Economic Review*, 42:4, 847-869.

[14] Wallace, N. and T. Zhu, 2004, A Commodity-Money Refinement in Matching Models, *Journal of Economic Theory* 117, 246-258

[15] Zhou, R., 2003, Does Commodity Money Eliminate the Indeterminacy of Equilibria?, *Journal of Economic Theory* 110, 176–190.

[16] Zhu, T., 2003, Existence of a Monetary Steady State in a Matching Model: Indivisible Money, *Journal of Economic Theory* 112, 307-324.

[17] Zhu, T., 2005, Existence of a Monetary Steady State in a Matching Model: Divisible Money, *Journal of Economic Theory* 123, 135-160.

A Proofs

Proof of Lemma 1 First, note that, if all the other agents are following a cut-off strategy at z^*, then the optimal strategy only depends on the current state of the economy, i.e., it does not depend on the particular history of realizations that led up to that state. This is so because Δz_t follows a continuous probability distribution that is independent of the current and the past states of the economy. Let then $\sigma^*(z^*)$ denote the optimal strategy of an agent given that all other agents are following a cut-off strategy at z^*. If the current state is z and the agent follows the optimal strategy, we let the expected payoff of holding one unit of money at the end of the period be given $v_1(z, z^*)$, and the expected payoff of holding zero units of money at the end of the period be given by $v_0(z, z^*)$. The optimal strategy can thus be described by the set $Z^* = \{z \in \mathbb{R} | -c + v_1(z, z^*) > v_0(z, z^*)\}$, i.e., the set of all states in which it is optimal to accept money. If we let $\Delta v(z, z^*) = v_1(z, z^*) - v_0(z, z^*)$, our objective is to prove that $\Delta v(z, z^*)$ is strictly increasing in z.

In what follows, we will first ignore the intrinsic utility or disutility of money by assuming $\widehat{z} = \infty$. We will return to this point at the end. Let z' be the state of the economy in period 1 and let $\sigma_w(z^*)$ be the following (potentially sub-optimal) strategy: (1) accept money in state z' if and only if $z' + w \in Z^*$, where $w > 0$, (2) if the agent participated in a meeting in which money was acceptable by at least one agent in a previous period, then follow the optimal strategy from that period on, (3) if the agent did not participate in a meeting where money was acceptable by at least one agent in a previous period, then accept money if and only if either $z' + w + \Delta z_1 \in Z^*$ or $z' + w + \Delta z_1 > z^*$, where Δz_1 is the realization of the stochastic process in period 1, (4) if the agent did not participate in a meeting where money was acceptable by at least one agent in a previous period, and it was not the case that $z' + w + \Delta z_1 \in Z^*$ or $z' + w + \Delta z_1 > z^*$, then behave as in (3) replacing $z' + w + \Delta z_1$ with $z' + w + \Delta z_1 + \Delta z_2$, where Δz_2 is the realization of the stochastic process in period 2. This process continues until a period is reached where the agent participates in a meeting where money is acceptable by at least one of the agents in the meeting, in which case the agent follows the optimal strategy from that period on.

Note that, if the agent following $\sigma_w(z^*)$ accepts money in a given period, his expected payoff in state z is given by $-c + v_1(z, z^*)$. Indeed, the agent follows the optimal strategy after accepting money. Let then $V_n(z, z^*)$ be the expected payoff of not accepting money in state z if the agent follows the strategy $\sigma_w(z^*)$. Define $\Delta V(z'; z^*) = v_1(z', z^*) - V_n(z', z^*)$. Since $\sigma_w(z^*)$ is not necessarily the optimal strategy, $V_n(z', z^*) \leq v_0(z', z^*)$. Hence $\Delta V(z'; z^*) > \Delta v(z'; z^*)$. If we prove that $\Delta v(z' + w, z^*) > \Delta V(z', z^*)$, then it must be that $\Delta v(z' + w, z^*) > \Delta v(z', z^*)$, in which case $v_1(z, z^*) - v_0(z, z^*)$ is strictly increasing in z.

In order to prove that $\Delta v(z' + w, z^*) > \Delta V(z', z^*)$, we compare payoffs from an agent starting at z' following the strategy $\sigma_w(z^*)$ and another agent starting at $z' + w$ following the optimal strategy $\sigma^*(z^*)$, for any given realization of the random process Δz. There are six cases that we need to consider. Henceforth, let $DV(\Delta z)$ denote the difference between the net benefit of carrying money if the current state is $z' + w + \Delta z$ and the agent follows the optimal strategy, and the net benefit of carrying money if the current state is $z' + \Delta z$ and the agent follows $\sigma_w(z^*)$.

The first possibility is that $z' + \Delta z > z^*$ and $z' + w + \Delta z \in Z^*$. In this case, it is optimal to accept money if the current state is $z' + w + \Delta z$ (hence, money is accepted under $\sigma_w(z^*)$), and money is accepted by all other agents if the current state is either $z' + \Delta z$ or $z' + w + \Delta z$. This implies that $DV(\Delta z) = 0$.

The second possibility is that $z' + \Delta z < z^* < z' + w + \Delta z$ and $z' + w + \Delta z \in Z^*$. In this case, it is optimal to accept money if the current state is $z' + w + \Delta z$, money is accepted by all other agents if the state is $z' + w + \Delta z$, but is is not accepted by the other agents if the state is $z' + \Delta z$. This implies that

$$DV(\Delta z) = (1 - m)\left[u - \beta E_{z'+\Delta z}(v_1 - v_0)\right],$$

where $E_{z'+\Delta z}(v_1 - v_0)$ is the net continuation payoff of having money when the current state is $z' + \Delta z$ and the agent follows the optimal strategy.

The third possibility is that $z' + w + \Delta z < z^*$ and $z' + w + \Delta z \in Z^*$. In this case, it is optimal to accept money if the current state is $z' + w + \Delta z$, but money is not accepted by all other agents if the current state is either $z' + w + \Delta z$ or $z' + \Delta z$. This implies that

$$DV(\Delta z) = (1 - m)\beta\left[E_{z'+w+\Delta z}(v_1 - v_0) - E_{z'+\Delta z}(v_1 - v_0)\right].$$

The fourth possibility is that $z' + \Delta z > z^*$ and $z' + w + \Delta z \notin Z^*$. In this case, it is not optimal to accept money if the current state is $z' + w + \Delta z$ (hence, money is not accepted under $\sigma_w(z^*)$), and money is accepted by all other agents if the current state is either $z'' + \Delta z$ or $z' + \Delta z$. This implies that

$$DV(\Delta z) = m\beta\left[E_{z'+w+\Delta z}(v_1 - v_0) - E_{z'+\Delta z}(v_1 - v_0)\right].$$

The fifth possibility is that $z' + \Delta z < z^* < z' + w + \Delta z$ and $z' + w + \Delta z \notin Z^*$. In this case, it is not optimal to accept money if the current state is $z' + w + \Delta z$, money is accepted by all other agents if the current state is $z' + w + \Delta z$, but it is not accepted by the other agents if the current state is $z' + \Delta z$. This implies that

$$DV(\Delta z) = (1 - m)\left[u - \beta E_{z'+\Delta z}(v_1 - v_0)\right] + m\beta\left[E_{z'+w+\Delta z}(v_1 - v_0) - E_{z'+\Delta z}(v_1 - v_0)\right].$$

Finally, the last possibility is that $z' + w + \Delta z < z^*$ and $z' + w + \Delta z \notin Z^*$. In this case, irrespective of whether the current state is $z' + w + \Delta z$ or $z' + \Delta z$, money is not accepted by the agent and by all other agents. Thus

$$DV(\Delta z) = \beta\left[E_{z'+w+\Delta z}(v_1 - v_0) - E_{z'+\Delta z}(v_1 - v_0)\right].$$

Now, note that in all possibilities considered above, it is never the case that the agent following $\sigma_{z'}$ in state $z' + \Delta z$ obtains a higher flow payoff than the agent following the optimal strategy in state $z' + w + \Delta z$. Indeed, in the first, third, fourth and sixth possibilities, the flow payoff is the same for both agents. In turn, in the second and the fifth possibilities, the agent that follows the optimal strategy obtains a strictly higher flow payoff. This is so because $E_{z'+\Delta z}(v_1 - v_0)$ is bounded above by u. Now, since the agent's expected payoff is the expected sum of the flow payoffs he will obtain in the current and in all future periods, it must be the case that

$$\int_{\Delta z} DV(\Delta z) f(\Delta z) d\Delta z > 0.$$

This proves that $\Delta v(z' + w, z^*) > \Delta V(z', z^*)$, and so $\Delta v(z' + w, z^*) > \Delta v(z', z^*)$. Since $w > 0$ is generic, we obtain that $v_1(z, z^*) - v_0(z, z^*)$ is strictly increasing in z. This implies that the best response to all the other agents following a cut-off strategy at z^*, is to also follow a cut-off strategy. It also implies the the best response is unique, in which case we can denote it by the cut-off state $Z_\infty(z^*)$.

Now, let's consider the case with a finite $\widehat{z}$. As long as $Z_\infty(z^*) \in [-\widehat{z}, \widehat{z}]$, nothing changes in the proof, because the utility (or disutility) $\gamma(z)$ has no effect on the optimal strategy in the dominant regions (the agent finds it optimal to accept money if $z > \widehat{z}$ and not to accept money if $z < -\widehat{z}$) and for a small enough w, the agent following the strategy $\sigma_w(z^*)$ is never forced to accept money for some $z < -\widehat{z}$. The cut-off strategy for a finite $\widehat{z}$ is not affected by the dominant regions, $Z(z^*) = Z_\infty(z^*)$.

If $Z_\infty(z^*) > \widehat{z}$, the proof is not affected. If $v_1(z, z^*) - v_0(z, z^*)$ happens to be positive for all values of z, then $Z(z^*) = \widehat{z}$, and money is never accepted in the interesting region. Things are different in case $Z_\infty(z^*) < -\widehat{z}$. The proof cannot be adapted to cover this case because the agent following the strategy $\sigma_w(z^*)$ would be forced to accept money, and thus face the disutility cost $\gamma(z)$, in some states in which money has a negative intrinsic utility. Now, we know that ignoring the dominant regions, $Z_\infty(z^*) < -\widehat{z}$ implies that $v_1(z, z^*) - v_0(z, z^*) > 0$ for all $z \in [-\widehat{z}, \widehat{z}]$. We can show that once we impose that the agent cannot choose to make an effort in a set of arbitrary states, the difference $v_1(z, z^*) - v_0(z, z^*)$ can only increase. The argument runs as follows. The difference $v_1(z, z^*) - v_0(z, z^*)$ is given by

$$\sum_{t=0}^{\infty} \zeta_1(t) c \left(-1 + \beta^t\right) + \sum_{t=0}^{\infty} \zeta_2(t) \beta^t \left[(1 - m)u + mc\right]$$

where $\zeta_1(t)$ is the probability that an agent that has not accepted money at state z will accept money after t periods before the economy reaches a state $z > z^*$, and $\zeta_2(t)$ is the probability that the economy will reach a state $z > z^*$ before the agent that has not accepted money at state z will get another opportunity to accept money. The term multiplying $\zeta_1(t)$ is the cost of incurring the cost c at the current period instead of doing so after t periods. The term multiplying $\zeta_2(t)$ is the difference of the value of reaching a state $z > z^*$ with and without money – the argument here is the same as the one that leads to (7). Taking the option of accepting money of an agent in a later period can only decrease $\zeta_1(t)$ and increase $\zeta_2(t)$, which can only increase $v_1(z, z^*) - v_0(z, z^*)$. Hence $Z_\infty(z^*) < -\widehat{z}$ implies that the agent follows a cut-off strategy around the state $Z(z^*) = -\widehat{z}$.

Proof of Lemma 2 We want to show that, if an agent is following a cut-off strategy at $z \in [-\widehat{z}, \widehat{z}]$ and all the other agents are following a cut-off strategy at $z' \in [-\widehat{z}, \widehat{z}]$, then the net expected payoff of producing in exchange for money in state z is strictly decreasing in z'.

Consider, first, the scenario where $z' < z$. There are three possibilities that we need to consider, depending on the realization of the random process Δz. The first possibility is that $z + \Delta z < z'$. In this case, money is not accepted by the agent and is not accepted by all the other agents. Thus, the net expected payoff of having one unit of money is given by

$$\beta E_{z+\Delta z}[V_1(z + \Delta z, z', z) - V_0(z + \Delta z, z', z)].$$

The second possibility is that $z + \Delta z \in [z', z)$. In this case, the agent does not accept money but all the other agents do so. Thus, the net expected payoff of having one unit of money is given by

$$(1 - m)u + m\beta E_{z+\Delta z}[V_1(z + \Delta z, z', z) - V_0(z + \Delta z, z', z)].$$

Finally, the last possibility is that $z + \Delta z \geq z$. In this case, money is always accepted, and the net expected payoff of having one unit of money is

$$(1 - m)u + mc.$$

Note that, whenever $z' < z$, the probability of the last possibility is independent of z'. Now, since (i) the net expected payoff under the second possibility is higher than that of the first possibility, and (ii) the probability of the second possibility is strictly decreasing in z', it must be the case that the net benefit of producing in exchange for money in state z is strictly decreasing in z', for all $z' < z$.

Consider, now, the scenario where $z' > z$. Again, there are three possibilities that we need to consider, depending on the realization of the random process Δz. The first possibility is that $z + \Delta z < z$. In this case, since money is not accepted by the agent and is not accepted by all the other agents, the net expected payoff of having one unit of money is given by

$$\beta E_{z+\Delta z}[V_1(z + \Delta z, z', z) - V_0(z + \Delta z, z', z)].$$

The second possibility is that $z + \Delta z \in [z, z')$. In this case, the agent accepts money but all the other agents do not do so. As a result, the net expected payoff of having one unit of money is given by

$$(1 - m)\beta E_{z+\Delta z}[V_1(z + \Delta z, z', z) - V_0(z + \Delta z, z', z)] + mc.$$

The third possibility is that $z + \Delta z \geq z'$. In this case, money is always accepted, and the net expected payoff of having one unit of money is

$$(1 - m)u + mc.$$

Note that, whenever $z' > z$, the probability of the first possibility is independent of z'. Now, since (i) the net expected payoff under the second possibility is strictly lower than that of the third possibility ($\beta E_{z+\Delta z}[V_1(z+\Delta z, z', z) - V_0(z+\Delta z, z', z)]$ is bounded above by u), and (ii) the probability of the second possibility is strictly increasing in z', it must be the case that the net benefit of producing in exchange for money in state z is strictly decreasing in z', for all $z' > z$.

Proof of Lemma 3 First, since the random process Δz_t follows a continuous probability distribution that is independent of the current state z, it must be that for all z and z' in $[-\widehat{z}, \widehat{z}]$,

$$V_1(z,z,z) - V_0(z,z,z) = V_1(z',z',z') - V_0(z',z',z'). \tag{10}$$

Now, there exists a unique $Z(z)$ that is a best response to all agents following a cut-off strategy at z (Lemma 1). By definition, $Z(z)$ satisfies

$$\beta \left\{ V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right] \right\} = c.$$

Let $z \in (-\widehat{z}, \widehat{z})$ and assume that

$$\beta \left[V_1(z,z,z) - V_0(z,z,z) \right] > c. \tag{11}$$

This implies

$$-c + \beta \left\{ V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right] \right\} < -c + \beta \left[V_1(z,z,z) - V_0(z,z,z) \right],$$

that is,

$$V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right] < V_1(z,z,z) - V_0(z,z,z). \tag{12}$$

Given (10), we can rewrite (12) as

$$V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right] < V_1\left[Z(z), Z(z), Z(z)\right] - V_0\left[Z(z), Z(z), Z(z)\right].$$

Since $V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right]$ is strictly decreasing in z (Lemma 2), it must be that $Z(z) < z$. This implies that it is strictly optimal to accept money in state z. Since this reasoning holds for all $z \in (-\widehat{z}, \widehat{z})$, a continuity argument implies that is it also optimal to accept money in state $-\widehat{z}$.

Now let $z \in (-\widehat{z}, \widehat{z})$ and assume that

$$\beta \left[V_1(z,z,z) - V_0(z,z,z) \right] < c. \tag{13}$$

This implies

$$-c + \beta \left\{ V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right] \right\} > -c + \beta \left[V_1(z,z,z) - V_0(z,z,z) \right],$$

that is,

$$V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right] > V_1(z,z,z) - V_0(z,z,z). \tag{14}$$

Again, given (10), we can rewrite (14) as

$$V_1[Z(z), z, Z(z)] - V_0[Z(z), z, Z(z)] > V_1[Z(z), Z(z), Z(z)] - V_0[Z(z), Z(z), Z(z)].$$

Since $V_1[Z(z), z, Z(z)] - V_0[Z(z), z, Z(z)]$ is strictly decreasing in z, it must be that $Z(z) > z$. This implies that it is strictly optimal not to accept money in state z. Since this reasoning holds for all $z \in (-\widehat{z}, \widehat{z})$, a continuity argument implies that is it also optimal not to accept money in state $\widehat{z}$.

Proof of Lemma 4 Suppose an equilibrium where agents follow different thresholds $z_i^* \in [-\hat{z}, \hat{z}]$. Consider the highest and smallest thresholds, z_H^* and z_L^* respectively. Since agents have to be indifferent at those thresholds, we have

$$-c + \beta V_1(z_H^*, z_H^*) - \beta V_0(z_H^*, z_H^*) = 0 = -c + \beta V_1(z_L^*, z_L^*) - \beta V_0(z_L^*, z_L^*) \tag{15}$$

where $V_i(z', z')$ is the relative payoff of finishing the period with i units of money at state z', following a cut-off strategy at state z'. Now note that at the left of z_H^*, money is accepted with probability 1, while at the right of z_H^*, money is accepted with some probability between 0 and 1. Likewise, at the right of z_L^*, money is accepted with probability 0, while at the right of z_L^*, money is accepted with some probability between 0 and 1.

Now consider an agent at z_H^* following a cut-off strategy at z_H^* and an agent at z_L^* following a cut-off strategy at z_L^*. Suppose the realization of Δz is positive. Then, both agents are willing to accept money. With some probability, both would be able to spend money in which case the difference between having and not having money is worth

$$(1-m)u + mc$$

but with some probability only the agent that started at z_H^* is able to spend money, in which case the difference between having money or not for this agent is still $(1-m)\,u + mc$, but the difference between having money or not for the agent that started at z_L^* is worth

$$(1-m)\beta E_z(V_1 - V_0) + mc < (1-m)u + mc$$

Now suppose the realization of Δz is negative. Then they are not willing to accept money. With some probability, the agent starting at z_H^* is able to spend money, in which case the difference between the value having money or not for him is

$$(1-m)u + m\beta E_z(V_1 - V_0)$$

but for the agent starting at z_L^*, the relative value of having money is

$$\beta E_z(V_1 - V_0)$$

With some probability, nobody will be able to either accept or spend money, so the relative value of having money will be β times the value they will get in the following period. In all cases, whenever agents realize different payoffs, the relative value of having money for the agent at z_H^* is larger than the value for the agent at z_L^*, which means (15) cannot hold.

B The case of a discrete state space

In what follows we consider the case in which the state space is discrete, given by $\mathbb{Z}$, and the random process Δz_t follows a discrete probability distribution that is independent of z and t, with expected value $\mathrm{E}(\Delta z) \approx 0$ and variance $\mathrm{Var}(\Delta z) > 0$. In order to consider in a tractable manner both the region of parameters where agents coordinate in the use of money and the region where agents do not coordinate in the use of money, we simplify the the description of the environment by assuming that, in states $z > \widehat{z}$, it is strictly optimal to accept money because a special agent in the economy forces other agents to do so.

First, it is straightforward to adapt the proofs of Lemmas 1 and 2 to the discrete case. Thus, as in Lemma 1, if all the other agents are following a cut-off strategy at some state $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$, in which money is accepted in state $z' \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$ if and only if $z' \geq z$, then there exists a unique best reply, given by the cut-off strategy $Z(z) \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$. Moreover, as in Lemma 2, the net benefit of following a cut-off strategy at some state $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$ is strictly decreasing in $z' \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$, where z' denotes the cut-off strategy followed by all the other agents. Combining Lemmas 1 and 2, it is also straightforward to show that Lemma 3 still holds, that is, in order to assess the agent's optimal behavior in some state $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$, it is sufficient to consider the payoffs induced by a symmetric strategy profile where all agents choose the same cut-off strategy z.

The only difference in the case of a discrete state space rests in the implications of Lemma 3. The reasoning runs as follows. As in the case of a continuous state space, since the random process Δz_t follows a probability distribution that is independent of the current state z, it must be that for all z and z' in $[-\widehat{z}, \widehat{z}]$,

$$V_1(z, z, z) - V_0(z, z, z) = V_1(z', z', z') - V_0(z', z', z').$$

First, assume that $\beta \left[V_1(z, z, z) - V_0(z, z, z)\right] > c$. There exists a unique $Z(z)$ that is a best response to all the other agents following a cut-off strategy at z (adaptation of Lemma 1). By definition, $Z(z)$ is the smallest $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$ such that

$$\beta \left\{ V_1 \left[Z(z), z, Z(z)\right] - V_0 \left[Z(z), z, Z(z)\right] \right\} \geq c.$$

Since $V_1 \left[Z(z), z, Z(z)\right] - V_0 \left[Z(z), z, Z(z)\right]$ is strictly decreasing in z (adaptation of Lemma 2), there are two possibilities: either $Z(z) = z$ or $Z(z) < z$. In the latter case, the same reasoning applied to the continuous state space implies that the unique symmetric equilibrium in cut-off strategies is the one in which all agents follow a cut-off strategy at $z = -\widehat{z}$. If, instead, $Z(z) = z$, then there exists a symmetric equilibrium in which all agents follow a cut-off strategy at z. Now, since z is generic, then there exists one such equilibrium for each $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$.

In order to pin down the whole region of parameters in which the unique equilibrium has money always accepted in the region $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$, we only need to consider the problem of an agent in state $z - 1 \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$, who follows a cut-off strategy at this state and believes that all the other agents are following a cut-off strategy at state $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$. This agent has an strict incentive to produce in exchange for money if and only if

$$\beta \left[V_1(z-1, z, z-1) - V_0(z-1, z, z-1)\right] > c.$$

Denote by $\phi(t)$ the probability of reaching any state strictly larger than $z-1$ at time $s+t$ and not before. Then, we can rewrite the above condition as

$$\left(\sum_{t=1}^{\infty} \beta^t \phi\left(t\right)\right) \left[\left(1-m\right) u + mc\right] > c. \tag{16}$$

Now, assume that $\beta \left[V_1(z, z, z) - V_0(z, z, z)\right] < c$. Again, there exists a unique $Z(z)$ that is a best response to all the other agents following a cut-off strategy at z. By definition, $Z(z)$ is the largest $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$ such that

$$\beta \left\{V_1\left[Z(z)-1, z, Z(z)-1\right] - V_0\left[Z(z)-1, z, Z(z)-1\right]\right\} \leq c.$$

Since $V_1\left[Z(z), z, Z(z)\right] - V_0\left[Z(z), z, Z(z)\right]$ is strictly decreasing in z, there are two possibilities: either $Z(z) = z$ or $Z(z) > z$. In the latter case, there exists a unique symmetric equilibrium in cut-off strategies, in which all agents follow a cut-off strategy at $z = \widehat{z} + 1$. If, instead, $Z(z) = z$, then there exists a symmetric equilibrium in which all agents follow a cut-off strategy at $z + 1$. Again, since z is generic, then there exists one such equilibrium for each $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$.

In order to pin down the whole region of parameters in which the unique equilibrium has money never accepted in the region $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$, we only need to consider the problem of an agent in state $z-1$ who follows a cut-off strategy at state $z \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$ and believes that all the other agents are following a cut-off strategy at state $z-1 \in \mathbb{Z} \cap [-\widehat{z}, \widehat{z}]$. This agent has an strict incentive not to produce in exchange for money if and only if

$$\beta \left[V_1(z-1, z-1, z) - V_0(z-1, z-1, z)\right] < c.$$

Denote by $\phi_+(t)$ the probability of reaching any state larger or equal than $z-1$ at time $s+t$ and not before. Then, we can rewrite the above condition as

$$\left(\sum_{t=1}^{\infty} \beta^t \phi_+\left(t\right)\right) \left[\left(1-m\right) u + mc\right] < c. \tag{17}$$

Equations (16) or (17) are versions of (2) with $\phi(t)$ or $\phi_+(t)$ instead of $\varphi\left(t\right)$. In the continuous case we consider the probabilities of reaching z when the economy starts in a state that is arbitrarily close to z. Here, we have to start from the closest state where money is not accepted or the closest state where money is accepted, depending on which strategies we want to eliminate. As there is some distance between them, the probabilities $\phi\left(t\right)$ and $\phi_+\left(t\right)$ will not be the same. Hence there will be a region with multiple equilibria. But as the support of Δz increases, the discrete distribution gets closer to a continuous distribution, and $\phi(t)$ and $\phi_+(t)$ get closer and closer to each other.

As in the continuous case, we can rewrite condition (16) as

$$\lambda_M \beta \left[(1-m)\, u + mc\right] > c,$$

and condition (17) as

$$\lambda_A \beta \left[(1-m)\, u + mc\right] < c,$$

where

$$\lambda_M = \sum_{t=1}^{\infty} \beta^{t-1} \phi(t) \quad \text{and} \quad \lambda_A = \sum_{t=1}^{\infty} \beta^{t-1} \phi_+(t) \tag{18}$$

The benchmark case corresponds to $\lambda_M = 0$, which means autarky is always an equilibrium, and $\lambda_A = 1$, which means money is an equilibrium as long as it is an equilibrium in the benchmark model. Clearly, (18) shows that λ_M and λ_A are increasing in β. Moreover, If $\mathrm{E}(\Delta z) = 0$, $\sum_{t=1}^{\infty} \phi_+(t) = 1$ and $\sum_{t=1}^{\infty} \phi(t) = 1$, thus as $\beta \to 1$, $\lambda_A \to 1$ and $\lambda_M \to 1$. That is, as the frequency of trade meetings goes to infinity, agents always coordinate in the use of money.

B.1 Example: Binary case

Consider a simple stochastic process where $\Pr(\Delta z = 1) = p$ and $\Pr(\Delta z = -1) = 1 - p$. Departing from state $z^* - 1$ in period s, the probability of reaching state z^* in period $s+1$ is p. Otherwise, the economy moves to state $z^* - 2$. Then, state z^* can only be reached in period $s+3$. The probabilities that state z^* will be reached for the first time at time $s+t$ are given by (for all $i \geq 0$)

$$\phi(2i+1) = \frac{(2i)!}{i!\,(i+1)!} p^{i+1} (1-p)^i \quad \text{and} \quad \phi(2i) = 0.$$

Remember that $\phi(t)$ is the probability of reaching for the first time state z^* at time $s+t$, when the initial state is $z^* - 1$. The formula for $\phi(2i+1)$ resembles a binomial distribution, but the usual combination is replaced with the Catalan numbers.[16] The value of λ_M is given by

$$\lambda_M = \sum_{i=0}^{\infty} \beta^{(2i)} \left(\frac{(2n)!}{n!\,(n+1)!} p^{i+1} (1-p)^i \right). \tag{19}$$

Departing from state z^* in period s, the probability of reaching a state larger than z^* in period $s+1$ is p. Otherwise, the economy moves to state $z^* - 1$, which happens with probability $1-p$. At $z^* - 1$, we are at the previous case. Thus (for all $i \geq 0$)

$$\begin{aligned} \phi_+(1) &= p, \\ \phi_+(2i+2) &= (1-p) \frac{(2i)!}{i!\,(i+1)!} p^{i+1} (1-p)^i, \\ \phi_+(2i+3) &= 0. \end{aligned}$$

[16] See, e.g., http://mathworld.wolfram.com/CatalanNumber.html.

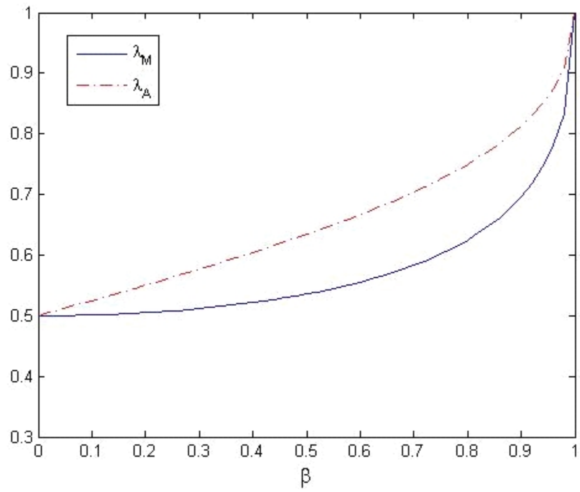

Figure 3: Binary case: λ_M and λ_A

Hence

$$\begin{aligned}\lambda_A &= p + (1-p)\sum_{i=0}^{\infty}\beta^{(2i+1)}\left(\frac{(2n)!}{n!\,(n+1)!}p^{i+1}(1-p)^i\right)\\ \lambda_A &= p + (1-p)\beta\lambda_M \qquad (20)\end{aligned}$$

The case $p = 0.5$ If $p = 0.5$, λ_M becomes:

$$\lambda_M = \sum_{i=0}^{\infty}\beta^{(2i)}\left(\frac{(2i)!}{i!\,(i+1)!}\left(\frac{1}{2}\right)^{2i+1}\right) \qquad (21)$$

which is a function of β only, and λ_A is then $\lambda_A = \frac{1+\beta\lambda^M}{2}$. Figure 3 shows λ_A and λ_M as a function of β. It turns out that the factor λ for a normal distributions lies between the lines for λ_A and λ_M. The multiple-equilibrium region is larger for intermediate values of β, and vanishes as β approaches 1. The curves for λ_A and λ_M are similar to λ in the continuous case: both λ_M and λ_A converge to 1 as β approaches 1, converge to 0.5 as β approaches 0, are increasing in β and convex.